NOISE

In Gravitational Wave Detectors
• One of the main battles for any precision instrument is to **identify** various noise sources and try to develop techniques to **reduce** the them
Familiar Laser interferometer noise curve
WE DID IT!
Diagnosis—finding the noise sources
Simplified sensitivity curves —without the narrow banc peaks

1. LIGO I total
2. Filtered seismic noise
3. Suspension thermal noise
4. Internal thermal noise - sapphire
5. Internal thermal noise - fused silica (fallback)
6. Shot noise
7. Radiation pressure noise
8. LIGO II total
Limiting noise sources—so far

- Seismic noise
- Thermal noise
  - Suspension thermal noise
- Quantum noise
  - Photon Shot Noise
- Seismic ‘cutoff’
  - High performance vibration isolators
- Newtonian background limit

- Suspension thermal noise
  - Very low loss suspension scheme
- Test mass thermal noise
  - Very low loss materials (sapphire, fused silica, silicon)
- Mirror Coating Noise

- Unified quantum noise limit
  - Optimized laser power
  - Beating quantum limit?
Quantum Noise
— simplified version

• Quantum uncertainty in the mirror position
  - Shot noise (Photon counting noise): Quantum fluctuation of the number of photons in the photo detector
  - Radiation pressure noise; Quantum fluctuation of the optical field that drive the mirror
Shot noise

- Shot noise in general term is the fluctuation of arriving time of signal carrier, could be electron or photon.

- In GW detector, photon is used to measure $x$

$$\Delta N_{\text{photon}} \rightarrow \Delta X$$
At the output of the interferometer, the number of photons arriving photodetector in a measurement time $\tau$

$$\langle N_{out} \rangle = N \sin^2 \left( \frac{\varphi}{2} \right)$$

(1)

$P_0 = N(\hbar \omega)$,

$N$—number of photons passing through the interferometer in time $\tau$; $\omega$ laser frequency

$\varphi = 2\pi x/\lambda$,

$x$—interferometer optical path difference
From statistics, the standard deviation of mean counts $N$ during $\tau$ is

$$\Delta N = \sqrt{N}$$

$$\Rightarrow \Delta N_{out} = \sqrt{N_{out}} = \sqrt{N \sin(\phi/2)} \quad (2)$$

We can also see that (from equation (1))

$$\delta N_{out} \sim N(\pi/\lambda) \sin(\phi) \delta x \quad (3)$$

$$\delta x_{shot} = \sqrt{\frac{\hbar c \lambda}{2\pi P_0}} \cos(\phi/2) \quad \text{m/}\sqrt{\text{Hz}}$$

- Shot noise is minimum when operate at dark fringe $\phi = 0$
- Shorter wavelength, Higher power $\Rightarrow$ low $\delta x_{shot}$

(That is not the reason we use infrared laser source, it is because it is the best so far)
Radiation Pressure noise

The force exerted on the mirror with power $P$ is

$$F = \frac{P_0}{c} = m\ddot{x} \quad x = X \sin(2\pi ft)$$

Recall:

$$P_0 \propto N(\hbar \omega)$$

From previous discussion, we can get:

$$x_{rp} = \frac{1}{mf^2} \sqrt{\frac{\hbar P_0}{8\pi^3 c \lambda}} \quad f \text{— frequency of the motion}$$

- High power $\rightarrow$ high $x_{rp}$
Standard Quantum Limit (SQL) —general

Quantum uncertainty in the mirror position

\[ \Delta x \Delta p \geq \hbar \]

\[ \Delta p \sim m \Delta x / \tau \]

\[ \Delta x > \sqrt{\hbar \tau / m} \]

\[ \Delta h = \Delta x / L \]

for \( \tau \sim 1 \text{ms}, \ m \sim 40 \text{kg} \rightarrow \Delta h \sim 10^{-23} \]
Standard Quantum Limit — interferometer

For GW detectors:

- $\Delta X_{\text{shot}}$ (phase) & $\Delta X_{\text{rp}}$ (amplitude) are a pair

- SQL is obtained when $\Delta X_{\text{shot}} = \Delta X_{\text{rp}}$
Some remarks

- The “flat” shot noise spectrum is for simple interferometer.

- For interferometer with cavities, the signal at high frequency at Photodetector is small, thus the rise of shot noise at high frequencies for GW detector sensitivity curves.

- Detailed analysis shown (may be discussed later at Chen’s lectures) that the quantum limit comes purely from light but not the quantisation of test mass.

- The SQL is the limit on displacement sensing in the absence of correlations between shot noise and radiation pressure noise.
Beating the Quantum Limit

• Inject squeezed state light to reduce the dominant noise in the appropriate quadrature

• Filter the signal before detection

• Create correlations between the radiation-pressure and the shot noise, such as using signal recycling mirror in the interferometer.
Assignment

• Derive radiation pressure noise

• Read the paper “Quantum Noise in Gravitational-wave Interferometers” by Thomas Corbitt and Nergis Mavalvala and write a short Summary about the methods of beating the quantum limit
Seismic Noise & Vibration Isolation Systems
Seismic noise

Ground noise:

\[ X = \alpha / f^2 \text{ (m/\sqrt{Hz})} \]

\( \alpha: 10^{-6} \sim 10^{-9} \)

@ \( f = 10 \text{ Hz}, \ x = 10^{-11} \text{ m} \)

GW detector requirement:

\( 10^{-18} \text{ m} @ 10 \text{ Hz} \)

→ Vibration isolation essential
Vibration isolation—basic ideas

Mechanical low pass filter

Below resonant
\[ f \ll f_0 \]
\[ x = x_0 \]
follow excitation

At resonant
\[ f = f_0 \]
\[ x > x_0 \]
motion amplified

Above resonant
\[ f \gg f_0 \]
\[ x > x_0 \]
vibration isolation
Vibration Isolators
—horizontal and vertical components

Vertical—Spring/Mass system

Horizontal—Pendulum

* Transfer function

\[ TF = \frac{output}{input} = \frac{x}{x_0} \]
Transfer Function—single stage

\[ \frac{X}{X_0} = (\frac{f_0}{f})^2 \]
Some numbers

- 1 Hz isolator, $TF@10\text{Hz} \approx (1/10)^2 = 0.01$
- @10 Hz, seismic $x_0 = 10^{-11}\text{m}$
- $x = x_0 \times TF = 10^{-14}\text{m}$ Not good enough

Multiple stages: $TF = TF_1 \times TF_2 \times TF_3 \times \ldots \times TF_N$ @ high frequency
Multi-stage isolator

Vertical isolator—spring/mass system

Horizontal isolator—pendulums

Integrated 3-D isolator

Roll off slope $\propto \left(\frac{f_0}{f}\right)^2\left(\frac{f_1}{f}\right)^2\left(\frac{f_2}{f}\right)^2\ldots\left(\frac{f_n}{f}\right)^2$

If $f_0=f_1=f_2=\ldots$, then slope $\propto \left(\frac{f_0}{f}\right)^{2n}$
Transfer function—multi-stage

5 stage 1 Hz pendulum

Now @10Hz
\[ TF = \left(\frac{1}{10}\right)^2 \times 5 \]
\[ X = x_0 \times TF = 10^{-21} \text{ m} \]

Wonderful
Some facts

• Coupled system: the total transfer function is not simply multiplication
  – At high frequency, TF=TF1xTF2xTF3…
  – Near resonant frequencies, the peaks spilt

• Corner frequency—highest resonant peak
  – Spring mass system with identical stages: $f_c \sim 2f_0$
  – Pendulum system with identical stages: $f_c \sim 2f_0 \sqrt{N}$
At low frequencies....

• Vibration isolation—mechanical low pass filter

• High resonant peaks at low frequency—amplitude $\propto$ Q-factor of the isolation components

• Hard for interferometer cavity locking control

• Needs damping to reduce the peaks height

• Or....
Pre-isolation stages

- Have an Ultra-low frequency (ULF) pre-isolation stage in front of the isolation chain with $f_0 \sim 0.1$ Hz. Then the peaks of the chain will fall on the slope of the this pre-isolator.
A single ultra ultra ultra low f stage?

Transfer function

Not as simple as that!
Internal frequencies

• Any structure will have internal resonances.

• Chladni figures are the results of the internal resonates of the plate.
Example: cantilever

<table>
<thead>
<tr>
<th>Mode</th>
<th>f</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>163.26</td>
</tr>
<tr>
<td>2</td>
<td>1016.7</td>
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<tr>
<td>3</td>
<td>1030.7</td>
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<tr>
<td>4</td>
<td>1498.1</td>
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<td>5</td>
<td>2835.7</td>
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<td>6</td>
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<td>7</td>
<td>5521.9</td>
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<tr>
<td>8</td>
<td>5743.3</td>
</tr>
</tbody>
</table>
Pre-isolation stage working range

• Low frequency pre-isolation is usually large structures with low internal frequency.

• Usually the first internal frequency is roughly 10~100 times of $f_0$.

• It roll-off does not go on forever.

• Needs to be combined with low frequency isolation chain.
Remarks

• Due to the cross coupling between the vertical and horizontal direction $\sim 10^{-3}$ (nothing is perfect), it is important to have good vertical isolation system.

• It is relatively easy to create horizontal low frequency isolation system—pendulums

• Vertical low frequency isolation system is not so straightforward (coil spring, cantilever…)
  – Needs to support heavy load—spring creeping
  – Low frequency $\rightarrow$ larger structure $\rightarrow$ lower internal modes
Pre-isolation

- Linkages
- Anti-springs
- Example: Inverted pendulum (wobbly table)

Gravity acted as “anti-spring”
UWA Vibration Isolation and Suspension System

Inverse Pendulum (Horizontal) 0.05 Hz

Roberts Linkage (Horizontal) 0.05 Hz

LaCoste (Vertical) 0.5Hz

4 horizontal Self-damped pendulum stages (~0.6Hz)

4 Vertical Euler Springs stage (~0.6Hz)

Test mass
Newtonian Noise - fundamental limit

Gravity Gradient Noise

Local gravitational field fluctuation:
- masses (people, vehicles, kangaroos) moving
- ground density change due to land wave.....
  (go underground)

Environment is coupled to test masses by an equivalent spring with typical frequency

\[ f_{\text{grav}} \approx \frac{\sqrt{G\rho}}{2\pi} \approx 7 \times 10^{-5} \text{ Hz} \]
Active Isolation systems

- The mass-spring / pendulum — passive (or soft) system

- Active (stiff) system
The motion of m is sensed and fed back to suspension point to reduce $x_0$ and thus $x$
Requirements for Active systems

• Complicated servo control

• Needs very good seismometer to sense the motion of $m$. Noise in seismometer would be taken as displacement signal and feedback

Remark: the passive isolation chain is actually a very good seismometer

• Usually the a isolation system is a combination of both but more active-like or more passive-like
LIGO isolation system

Active isolation platform + quadruple pendulum
Thermal Noise
Fluctuation-Dissipation Theorem

Brownian motion:

Fluctuations in the rate of impacts of individual water molecules on a grain

\[ \frac{x^2}{x_{\text{therm}}} = k_B T \frac{1}{3\pi a \eta} \tau \]

\( \tau \) – observation time
\( a \) – radius of a spherical grain
\( \eta \) – viscosity of the fluid

Link between fluctuation (microscopic phenomena) and dissipation (property of continuous system)
Fluctuation-Dissipation Theorem

The power spectrum of the minimal fluctuation force on a system:

\[ F_{\text{therm}}^2(f) = 4k_B T \Re(Z(f)) \]

- \( F_{\text{therm}}(f) \) — fluctuating force on a system
- \( R(z) \) — real (dissipative) part of impedance of the system

Power spectrum of the system’s fluctuating motion

\[ \chi_{\text{therm}}^2 = \frac{k_B T}{\pi^2 f^2} \Re(Z^{-1}) \]

- Reduce thermal noise:
  - Reduce dissipation
    — low loss material;
  - Reduce temperature
    — cryogenic detector
Loss & Loss Angle

\[ F = -k \delta x \]

For real materials, the stress-strain response is not instantaneous — anelasticity

\[ k(f) = k_0 (1 + i \phi(f)) \]

\( \phi(f) \) — loss angle, dissipation from internal friction in materials

In practice, \( \phi \sim \) independent of frequency or weekly dependent
The Quality Factor (Q-factor)

Definition:

\[ Q = \frac{E}{\Delta E} = \frac{f}{\Delta f} \]

\( \Delta f \)—width of the resonant peak at \( \frac{1}{2} \) power (\( 1/\sqrt{2} \) amplitude)

**Frequency domain**

\[ Q = \pi f_0 \tau = \pi f_0 \frac{(t-t_0)}{\ln \frac{A}{A_0}} \]

\( f_0 \)—resonant frequency; \( \tau \)—period

**Time domain**
Q-factor and thermal noise

• Each oscillation mode has $\sim k_B T$ thermal energy

• $Q=1/\phi$

• Low loss (high Q) system, this energy is concentrated at the resonant frequency. Thus the thermal noise at off resonant frequency is low.
Pendulum Thermal Noise

Restoring force =
Gravity (lossless) + elastic force (lossy) at the bending

$$k_{pen} = k'_e + k_{grav}$$

$$k'_e = k_e (1 + i\phi_e)$$

$$\phi_{pend} = \phi_{flex} \frac{k_e}{k_e + k_{grav}} \approx \phi_{flex} \frac{k_e}{k_{grav}}$$

Dilution factor

Pendulum loss is much smaller then the bending flexure loss!
Pendulum Thermal Noise Spectrum

\[ x^2(f) = \frac{4k_B T k_{pen} \phi_{pen}}{2\pi f \left[ (k_{pen} - m(2\pi f)^2)^2 + k_{pen}^2 \phi_{pen}^2 \right] } \]

Above \( f_p \) important.
Test Mass Internal Thermal Noise

Test mass has many internal resonant modes. Depends on the size of the test mass, the internal frequencies typically starts ~20kHz for test mass with several tens kg.

1. 17.4kHz
2. 17.4 kHz
3. 26.09 kHz
4. 28.4 kHz
5. 28.4 kHz
6. 31.7 kHz
Mode expansion

Treat the system as a superposition of independent harmonic oscillation

\[
x^2(f) = \sum \frac{4k_BT k_i \phi_i}{2\pi f [(k_i - m_i (2\pi f)^2)^2 + k_i^2 \phi_i^2]}
\]

Reduce \(\phi\)—low loss material;
reduce \(T\)—cryogenic detector
Thermal noise for long wire suspension

Frequency Hz

Displacement Noise m/Hz$^{1/2}$

Pendulum mode

violin string modes

Internal modes

Below $f_i$ spectrum
Test mass and suspension

- **Very low loss test masses**
  - Sapphire, fused silica, silicon (ringing for 30min at 50kHz, 10,000 cycles before the amplitude drop to 1/3)

- **Very low loss suspension**
  - Fused silica fibre, ribbon, niobium ribbon
    (pendulum swinging from months)
Remarks

• In the case of inhomogeneously distributed loss, the thermal noise spectrum may differ from that derived from mode expansion method.
  – Loss $\phi$ is a function of position $\phi(r)$. Inhomogeneity causes correlations between the fluctuations in the motions of different modes. There would be extra coupling terms in mode expansion expression.

• For interferometer, the thermal noise is sensed by the laser intensity profile at the middle of the test mass, not the whole surface of the test mass.
Thermal noise—direct approach (levin’s approach)

• The spectrum density of fluctuation readout variable $x(t)$ (due to test mass position fluctuation):

\[
S_x(f) = \frac{k_B T}{\pi^2 f^2} \frac{W_{\text{diss}}}{F_0^2}
\]

• Apply pressure $P = F_0 \cos(\omega t)f(r)$
• Calculate the average power dissipated $W_{\text{diss}}$ in the test mass
Calculate $W_{\text{loss}}$
Finite element modeling

$$W_{\text{loss}} = \omega \int E_{el}(\vec{r})\phi(\vec{r})dV$$

Maximum elastic strain under pressure
Direct Approach Results

• Agrees with mode expansion when loss is homogeneous

• Thermal noise due to surface losses near the laser beam spot

$$S_x(f) \propto \frac{1}{r_0^2}$$

• Thermal noise due to volume losses

$$S_x(f) \propto \frac{1}{r_0}$$

  – Small beam size, surface loss dominate
  – Very good coating required
  • Current limiting thermal noise source—intensive research on reducing coating noise
Direct Approach also results in:

- Lossy elements far from the laser spot or at the minimum strain place on the test mass do not contribute very much in total thermal noise level.
  - We can put “rubbish” on the test mass if required without degrading thermal noise much (bonding ears, damping ring, vibration absorbers...).
  - Position of the damping items are important!
Thermal elastic noise

- Due to random heat flow—Thermal gradient

\[ S(f) = \frac{8 k_B T^2 \alpha^2 (1 + \sigma)^2 \kappa}{\sqrt{2\pi} (\rho C)^2 r^3 \omega^3} \]

Sapphire is bad in this aspect: \( \kappa \sim 42, \alpha \sim 6 \times 10^{-6} \)
\( \rightarrow \) not negligible in room temperature detectors

Compared with fused silica: \( \kappa \sim 1.4, \alpha \sim 0.5 \times 10^{-6} \)

V.B. Braginsky, M.L. Gorodetsky, S.P. Vyatchanin, *Phys. Lett. A* 264 1 1999
Reducing thermoelastic noise

• Reduce the temperature gradient
  – Increase the beam size to (to a limit that the *diffraction loss* not too big)

  – Change beam shape—from Gaussian to flattop

![Diagram of Gaussian and Flattop beams]
Thermo-refractive noise

- **Multi-layer coating**

- **Temperature fluctuation causes the layer thickness to change and thus the reflected light phase.**

\[
S(f) = \frac{\sqrt{2}k_BT^2}{\pi r_0 \sqrt{\omega C \kappa}}
\]

*V.B. Braginsky et al., Physics Letters A 271 (2000) 303–307*
Assignment for vibration isolation and thermal noise

• Assuming a seismic noise with $\alpha=10^{-9} \text{ m/}\sqrt{\text{Hz}}$, for advanced LIGO detector, theoretically how many stages of 1 Hz pendulum is sufficient to meet the requirement before thermal noise dominate (Use the noise curve for the 4km interferometer in lecture 11, slide 8 for thermal noise estimation)? What would be the corner frequency of such a system?

• Read the attached article “lossangle” and write a paragraph of your understanding of the loss angle

• Derive the expression of pendulum and test mass thermal noise in terms of Q (use mode expansion approximation for test mass thermal noise). Give a expression for $f >> f_p$ for pendulum, and $f << f_i$ (use the lowest mode) for test mass. What will be the requirement of the pendulum Q and test mass Q for advanced GW detector?