Background of gravitational waves from pre-galactic black holes formation

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Abstract. We study the generation of a background of gravitational waves (GWs) produced from a Population of core-collapse supernovae, which form black holes in scenarios of structure formation. We discuss which astrophysical information could be obtained from a positive (or even a negative) detection of backgrounds of GWs generated in these scenarios. One of them is the possibility to obtain the initial and final redshifts of the emission period from the observed spectrum of GWs.

1. Introduction

Because of the fact that GWs are produced by a large variety of astrophysical sources and cosmological phenomena, it is quite probable that the universe is pervaded by a background of such waves. Binary stars of a variety of stars (ordinary, compact or combinations of them), Population III stars, phase transitions in the early universe, and cosmic strings are examples of sources that could produce such putative background of GWs (Thorne 1987).

In the present article we study the background of GWs produced from the population III black holes formation. The basic arguments in favor of the existence of these black holes are the following: a) the universe underwent a reheating (or reionization) phase between the standard recombination epoch (at $z \sim 1000$) and $z > 5$ (see, Haiman and Loeb 1997, and Loeb and Barkana 2001 for a review), as a result of the formation of the first structures of the Universe; b) the metalicity of $\sim 10^{-2} Z_{\odot}$ found in high $-\, z$ Lyα forest clouds (Songaila and Cowie 1996; Ellison *et al* 2000) is consistent with a stellar population formed at $z > 5$ (Venkatesan 2000).

The paper is organized as follows. In §2 we describe how to calculate the background of GWs produced during the formation of the black holes in the present scenario, in §3 we present the numerical results and the discussions, in §4 we discuss the detectability of this background, and the conclusions that could be drawn whether (or not) it were detected by the forthcoming GW observatories such as *LIGO* and *VIRGO*, and finally in §5 we present the conclusions.

2. The gravitational wave production

The dimensionless amplitude of the background of GWs, $h_{\rm BG}$, reads

$$h^2_{\rm BG} = \frac{1}{\nu_{\rm obs}} \int h^2_{\rm BH} dR_{\rm BH}, \tag{1}$$

(see, de Araujo *et al* 2000 for details) where $\nu_{\rm obs}$, $h_{\rm BH}$ and $dR_{\rm BH}$ are defined below.

The dimensionless amplitude $h_{\rm BH}$ produced by the collapse of a star, or star cluster, to form a black hole is (Thorne 1987)

$$h_{\rm BH} \simeq 7.4 \times 10^{-20} \varepsilon^{1/2}_{\rm GW} \left(\frac{M_{\rm r}}{\mathbf{M}_\odot}\right) \left(\frac{r_{\rm z}}{1{\rm Mpc}}\right)^{-1}, \tag{2}$$

where $\varepsilon_{\rm GW}$ is the efficiency of generation of GWs, $M_{\rm r}$ is the remnant black hole mass and $r_{\rm z}$ is the distance to the source. We assume that the black holes mass progenitor ranges from $M = 25 - 125{\rm M}_\odot$. The remnant and the progenitor masses are related to $M_{\rm r} = \alpha M$, and we assume $\alpha = 0.1$ (see, e.g., Ferrari *et al* 1999).

The collapse of a star to a black hole produces a signal with a observed frequency $\nu_{\rm obs}$ at the Earth (Thorne 1987)

$$\nu_{\rm obs} \simeq 1.3 \times 10^4 {\rm Hz} \left(\frac{{\rm M}_\odot}{M_{\rm r}}\right) (1+z)^{-1}, \tag{3}$$

where the factor $(1+z)^{-1}$ takes into account the redshift effect on the emission frequency.

The differential rate of black holes formation $dR_{\rm BH}$ reads

$$dR_{\rm BH} = \dot{\rho}_\star(z) \frac{dV}{dz} \phi(m) dm dz, \tag{4}$$

where dV is the comoving volume element, $\phi(m)$ is the stellar initial mass function (IMF), and $\dot{\rho}_\star(z)$ is the star formation rate (SFR) density.

The SFR density can be related to the reionization of the universe. The amount of baryons necessary to participate in early star formation, to account for the reionization, would amount to a small fraction, $f_\star$, of all baryons of the universe (see, e.g., Loeb and Barkana 2001). We then assume that

$$\dot{\rho}_\star \equiv \frac{d\rho_\star}{dt} = \frac{d}{dt}[\Omega_\star\ \rho_{\rm c}\ (1+z)^3], \tag{5}$$

where the term in brackets represents the stellar mass density at redshift z, with $\rho_{\rm c}$ the present critical density, and $\Omega_\star$ the stellar density parameter. The latter can be written as a fraction of the baryonic density parameter, namely, $\Omega_\star = f_\star \Omega_{\rm B}$.

From the above equations we obtain for the dimensionless amplitude

$$h^2_{\rm BG} = \frac{(7.4 \times 10^{-20} \alpha)^2 \varepsilon_{\rm GW}}{\nu_{\rm obs}} \times \left[\int_{z_{\rm cf}}^{z_{\rm ci}} \int_{m_{\rm min}}^{m_{\rm u}} \left(\frac{m}{{\rm M}_\odot}\right)^2 \left(\frac{d_{\rm L}}{1{\rm Mpc}}\right)^{-2} \dot{\rho}_\star(z) \frac{dV}{dz} \phi(m) dm dz\right], \tag{6}$$

where $d_{\rm L}$ is the luminosity distance to the source.

In the next section we present the numerical results and discussions, which come mainly from equation (6). Looking at this equation one notes that to integrate it, one needs to choose the IMF, the cosmological parameters, and to set values for the following parameters: $z_{\rm ci}$, $z_{\rm cf}$, α, $\varepsilon_{\rm GW}$, $f_\star$.

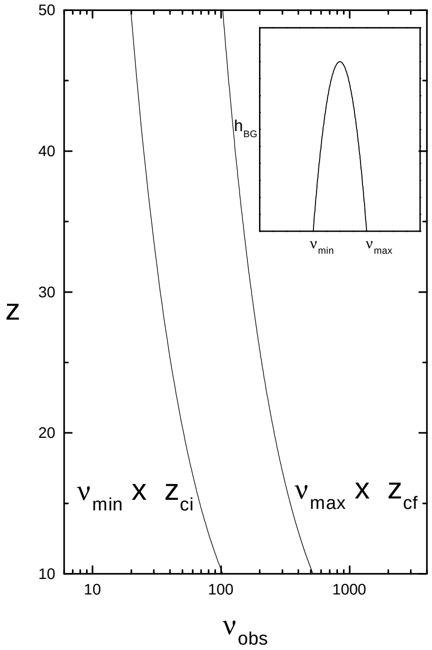

Figure 1. Example on how one could obtain from $h_{\rm BG}$ *vs.* $\nu_{\rm obs}$ the initial (final) redshift $z_{\rm ci}$ ($z_{\rm cf}$) of the GW emission period. We have adopted $\alpha = 0.1$.

3. Numerical results and discussions

To evaluate the background of GWs produced by the formation of the Population III black holes it is necessary to know in which redshifts they began and finished being formed. This is a very hard question to answer, since it involves the knowledge of the role of the negative and positive feedbacks of star formation which are regulated by cooling and injection of energy processes.

Should the stochastic background of GWs studied here be significantly produced and detected at a reasonable confidence level, the present study can be used to obtain the redshift range where the Population III black holes were formed, independently of any CDM modeling. In Figure 1 an example is given on how one could get $z_{\rm ci}$, and $z_{\rm cf}$ from the curve $h_{\rm BG}$ *vs.* $\nu_{\rm obs}$. Knowing the frequency band $\nu_{\rm min} - \nu_{\rm max}$ and using equation (3) one obtains $z_{\rm ci}$, $z_{\rm cf}$ (see Figure 1), which are, therefore observable. Note that we have assumed as Ferrari *et al* (1999) that α is a constant ($\alpha = 0.1$).

Stars start forming at different redshifts, creating ionized bubbles (Strömgren spheres) around themselves which expand into the intergalactic medium (IGM), at a rate dictated by the source luminosity and the background IGM density (Loeb and Barkana 2001). The reionization is complete when the bubbles overlap to fill the entire universe. Thus the epoch of reionization is not the epoch of star formation. There is a non negligible span of time between them. Here, we have chosen different formation

Table 1. The redshifts of collapse for our models and the corresponding GW frequency bands. The cosmological parameter $h_{100}^2 \Omega_{\rm B} = 0.019$ (see the text), $\alpha = 0.1$, $f_\star = 0.01$ (our fiducial value), and the standard IMF are adopted.

Model	$z_{\rm ci}$	$z_{\rm cf}$	$\Delta\nu_{\rm obs}$ (Hz)
A	20	10	50-470
B	30	20	34-250
C	40	30	25-170
D	50	40	20-130
E	30	10	34-470
F	40	10	25-470
G	50	10	20-470

epochs to see their influence on the putative background of GWs and, also to see if it could be detected by the forthcoming GW antennas.

To calculate $h_{\rm BG}$ we adopted the standard Salpeter IMF. For $\varepsilon_{\rm GW}$, the efficiency of production of GWs, whose distribution function is unknown, we have parameterized our results in terms of its maximum value, namely, $\varepsilon_{\rm GW_{max}} = 7 \times 10^{-4}$. This figure is obtained from studies by Stark and Piran (1986) who simulated the axisymmetric collapse of a rotating star to a black hole.

To calculate $h_{\rm BG}$ we still need to know $\Omega_\star$, which has a key role in the definition of the SFR density. From different studies one can conclude that a few percent, maybe up to $\sim 10\%$, of the baryons must be condensed into stars in order for the reionization of the universe to take place (see, e.g., Venkatesan 2000). Here we have set the value of $\Omega_\star$ in such a way that it amounts to 1% of all baryons (our fiducial value).

Looking at equation (6) one could think it would depend critically on the cosmological parameters. But our results show that $h_{\rm BG}$ depends only on H_0, and $\Omega_{\rm B}$, the Hubble parameter and the baryonic density parameter, respectively. The quantity $h_{100}^2 \Omega_{\rm B} = 0.019 \pm 0.0024$ (where h_{100} is the Hubble parameter given in terms of 100 km s^{-1} Mpc^{-1}) is obtained, for example, from Big Bang nucleosynthesis studies (see, e.g., Burles *et al* 1999).

In Table 1 we present the redshift band, $z_{\rm ci}$ and $z_{\rm cf}$ for the models studied and the corresponding GW frequency bands. For the cosmological parameters we have adopted $h_{100} = 0.65$, $\Omega_{\rm M} = 0.3$, $\Omega_{\rm B} = 0.045$ and $\Omega_\Lambda = 0.7$. We have also adopted $\alpha = 0.1$, $f_\star = 0.01$, and the standard IMF.

Note that no structure formation model has been used to find the black holes formation epoch, instead we have simply chosen the values of z to see whether it is possible to obtain detectable GWs signals.

A relevant question is whether the background we study here is continuous or not. The duty cycle indicates if the collective effect of the bursts of GWs generated during the collapse of a progenitor star generates a continuous background. For all the models studied here the duty cycle is $\gg 1$.

We find, for example, that the formation of a Population (III) of black holes, in the model D, could generate a stochastic background of GWs with amplitude $h_{\rm BG} \simeq (0.8{-}2)\times 10^{-24}$ and a corresponding closure density of $\Omega_{\rm GW} \simeq (0.7{-}1.4)\times 10^{-8}$, at the frequency band $\nu_{\rm obs} \simeq 20 - 130\,{\rm Hz}$ (assuming an efficiency of generation $\varepsilon_{\rm GW} \simeq 7 \times 10^{-4}$, the maximum one).

In other paper to appear elsewhere (de Araujo *et al* 2001) we study in detail how the variations of the several parameters modify our results.

Table 2. For the models of Table 1 we present the *SNR* for pairs of *LIGO* I, II and III ("first", "enhanced", and "advanced", respectively) observatories for 1 year of observation. Note that an efficiency of generation $\varepsilon_{\mathrm{GW}_{\max}} = 7 \times 10^{-4}$ is assumed.

	SNR		
Model	*LIGO* I	*LIGO* II	*LIGO* III
A	8.3×10^{-3}	1.6	6.6
B	8.5×10^{-3}	2.3	26
C	8.7×10^{-3}	2.7	47
D	8.1×10^{-3}	2.5	51
E	2.7×10^{-3}	5.7	37
F	5.0×10^{-3}	12	120
G	7.7×10^{-2}	21	260

4. Detectability of the background of gravitational waves

The background predicted in the present study cannot be detected by single forthcoming interferometric detectors, such as *VIRGO* and *LIGO* (even by the advanced one). However, it is possible to correlate the signal of two or more detectors to detect the background which we propose exists.

To assess the detectability of a GW signal one must evaluate the signal-to-noise ratio (*SNR*), which for a pair of interferometers is given by (see, e.g. Flanagan 1993)

$$SNR^2 = \left[\left(\frac{9H_0^4}{50\pi^4} \right) T \int_0^\infty d\nu \frac{\gamma^2(\nu)\Omega_{GW}^2(\nu)}{\nu^6 S_h^{(1)}(\nu) S_h^{(2)}(\nu)} \right] \tag{7}$$

where $S_h^{(i)}$ is the spectral noise density, T the integration time, and $\gamma(\nu)$ is the overlap reduction function, which depends on the relative positions and orientations of the two interferometers. The closure energy density is given by

$$\Omega_{\mathrm{GW}} = \frac{1}{\rho_{\mathrm{c}}} \frac{d\rho_{\mathrm{GW}}}{d\log\nu_{\mathrm{obs}}} = \frac{4\pi^2}{3H_0^2} \nu_{\mathrm{obs}}^2 h_{\mathrm{BG}}^2 . \tag{8}$$

Here we consider, in particular, the *LIGO* interferometers, and their spectral noise densities have been taken from a paper by Owen *et al* (1998).

In Table 2 we present the *SNR* for 1 year of observation with $\alpha = 0.1$, $\Omega_{\mathrm{B}} h_{100}^2 = 0.019$, $f_\star = 0.01$ and $\varepsilon_{\mathrm{GW}_{\max}} = 7 \times 10^{-4}$ for the models of Table 1, for the three *LIGO* interferometer configurations.

Note that for the "initial" *LIGO* (*LIGO* I) there is no hope of detecting the background of GWs we propose here. For the "enhanced" *LIGO* (*LIGO* II) there is some possibility of detecting the background, since $SNR > 1$, if $\varepsilon_{\mathrm{GW}}$ is around the maximum value. Even if the *LIGO* II interferometers cannot detect such a background it will be possible to constrain the efficiency of GW production.

The prospect for the detection with the "advanced" *LIGO* (*LIGO* III) interferometers is much more optimistic, since the *SNR* for almost all models is significantly greater than one. Only if the value of $\varepsilon_{\mathrm{GW}}$ were significantly lower than the maximum value the detection would be not possible. In fact, the signal to noise ratio is critically dependent on this parameter.

Note that the larger the star formation redshift band the greater the *SNR* is. Secondly, the earlier the star formation the greater the *SNR* is. It is worth recalling that if one can obtain the curve "$h_{\rm BG}$ *vs.* $\nu_{\rm obs}$" and if the value of α is known, one finds the redshift of star formation.

5. Conclusions

We have shown that a background of GWs is produced from population III black holes formation at high redshift. This background can in principle be detected by a pair of *LIGO* II (or most probably by a pair of *LIGO* III) interferometers. However, a relevant question should be considered: what astrophysical information can one obtain whether or not such a putative background is detected?

Firstly, let us consider a non detection of the background of GWs. The critical parameter to be constrained here is $\varepsilon_{\rm GW}$, a non detection would mean that the efficiency of GWs during the formation of black holes is not high enough. Another possibility is that the first generation of stars is such that the black holes formed had masses $> 100 M_{\odot}$, and should they form at $z > 10$ the GW frequency band would be out of the LIGO frequency band.

Secondly, a detection of the background with a significant SNR would permit us to obtain the curve $h_{\rm BG}$ *vs.* $\nu_{\rm obs}$. From it one can constraint α and the redshift formation epoch; and for a given IMF and $\Omega_{\rm B} h^2$, one can also constraint the values of $f_\star$ and $\varepsilon_{\rm GW}$. On the other hand, using the curve $h_{\rm BG}$ *vs.* $\nu_{\rm obs}$ and in addition other astrophysical data, say CBR data, models of structure formation and reionization of universe, constraint on $\varepsilon_{\rm GW}$ can also be imposed.

It is worth mentioning that a significant amount of GWs can also be produced during the formation of neutron stars, and if such stars are r-mode unstable. We leave these issues for another studies to appear elsewhere.

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