Gravitational radiation of nonlinear cosmic string

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Abstract. Any cosmic string is possible to consider as nonlinear if it performs nonlinear oscillations. These oscillations vary the properties of spacetime around a string drastically - it becomes nonstationary and depends on parameter of nonlinearity. Our estimations shown that the rate of gravitational energy radiation from nonlinear cosmic string could be essencially stronger then from linear one.

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1. Introduction

It is well known that cosmic strings were appeared at the very early stages of the Universe evolution. Cosmic string created by different types of scalar fields that were produced while the vacuum phase transitions, in its turn. One of lagrangians describes these fields is the Higgs's lagrangian for the complex scalar field $\chi(x)$ -

$$\mathcal{L} = \partial_\alpha \chi^* \partial^\alpha \chi + m^2 \chi^* \chi - \lambda (\chi^* \chi)^2. \tag{1}$$

This lagrangian is drastically nonlinear one (λ is a coupling constant describing the field self-interaction). That is why the cosmic strings, generally speaking, should be the nonlinear, also, i.e. should perform the nonlinear oscillations.

To describe them let us study the thread-like substance consists of the set of infinity thin crossless threads and full continiously given spacetime. The energy-momemtum tensor of this thread-like substance we may write down as

$$T^{\alpha\beta} = \mu u^\alpha u^\beta - t^{\alpha\beta}. \tag{2}$$

The second term can be interpret as the stress tensor appeared due to the oscillations of each thread. These oscillations occure in view of the string's tension or due to the elastic force. We should regard, in full analogy with classical mechanics [2], that elastic force include not only the linear term, but the nonlinear one, too. By viture of nonlinearity the stress tensor of thread-like substance has the following form

$$t^{\alpha\beta} = \mu l^\alpha l^\beta (1 - \frac{\varepsilon^2}{2} l^\gamma l_\gamma), \tag{3}$$

where ε is a dimensionless coefficient describing measure of nonlinearity. In this $\frac{\varepsilon^2}{2}$ is in order of λ, i.e. in order of 10^{-14}. It need be recalled that $u^\alpha = \dot{x}^\alpha = \frac{dx^\alpha}{d\tau}$ and $l^\alpha = \acute{x^\alpha} = \frac{dx^\alpha}{d\rho}$.

Introducing (2) and (3) into the energy-momentum conservational law we get the equation of motion of nonlinear cosmic string [3]

$$\dot{u}^\alpha - l^{\acute{\alpha}}\left(1 - \frac{\varepsilon^2}{2} l^\gamma l_\gamma\right) = 0. \tag{4}$$

The nonlinear character of cosmic string leads to appearance of some additional items in metric coefficients describing the spacetime interval in vicinity of such string.

2. Gravitational field of oscillating nonlinear cosmic string

In this section we will derive the metric of nonlinear cosmic string oscillating in the form of standing waves. We concentrate our attension on finding the nonlinear contributions to coefficients of metric tensor. For this purpose the energy-momentum tensor of nonlinear string-like substance is used

$$T^{\alpha\beta} = \mu\left[u^\alpha u^\beta - l^\alpha l^\beta (1 - \frac{\varepsilon^2}{2} l^\gamma l_\gamma)\right]. \tag{5}$$

The further calculations will be carried out in the assumption that gravitational field is weak, i.e. that the metric may be represented as $g_{\alpha\beta} = \delta_{\alpha\beta} + h_{\alpha\beta}$, where $h_{\alpha\beta}$ are small additions to the pseudo-Euclidian metric. Then in harmonical coordinates Einstein equations can be linearised

$$\Box h_{\alpha\beta} = -16\pi\gamma\Theta_{\alpha\beta}, \tag{6}$$

where

$$\Theta_{\alpha\beta} = T_{\alpha\beta} - \frac{1}{2}\delta_{\alpha\beta} T. \tag{7}$$

To calculate $\Theta_{\alpha\beta}$ it is necessary to take into account that vectors u^α and l^α obey the standard orthonormal gauge conditions - $u^\alpha u_\alpha + l^\alpha l_\alpha = 1; u^\alpha l_\alpha = 0$. Then from (5) for a solitary cosmic string we have

$$\Theta^{\alpha\beta} = \mu\left[u^\alpha u^\beta - l^\alpha l^\beta (1 - \frac{\varepsilon^2}{2} l^\gamma l_\gamma) + \delta^{\alpha\beta} l^\lambda l_\lambda (1 - \frac{\varepsilon^2}{4} l^\gamma l_\gamma)\right]\delta_3(x - x_0). \tag{8}$$

Let us impose now the supplemental restrictions on the form of spacetime interval. Namely, we shall look for it in the orthogonal form. In this case $l^0 = 0$. Besides, we shall suppose that cosmic string is motionless as a whole one, i.e. we shall accept that $u^k = 0$. Under these conditions the equation (6) with the right part (7) in components will be written down as follows

$$\Box h_{00} = 4\pi\gamma\mu\varepsilon^2\delta_{00}\delta_3(x - x_0), \tag{9}$$

$$\Box h_{kl} = 16\pi\gamma\mu\left[l_k l_l (1 + \frac{\varepsilon^2}{2} l^m l_m) + \delta_{kl} l^n l_n (1 + \frac{\varepsilon^2}{4} l^m l_m)\right]\delta_3(x - x_0), \tag{10}$$

i.e. they will accept the form of D'Alembert equation. Their solutions, as it is well-known, are the retarded potentials. Expanding them in power series of the parameter $\frac{|x - x'|}{x^0}$ and retaning only leading term we shall find the time-like component

$$h_{00} = -\gamma\mu\delta_{00}\int\limits_V \frac{\delta_3(x - x')}{|x - x'|} dV' = -2\gamma\mu\varepsilon^2 \ln\frac{r}{r_0}. \tag{11}$$

In order to find the space-like components it is necessary to define vector l^k. Hence, we have to solve nonlinear equation of motion in pseudo-Euclidian metric. Later on we shall use the approximation scheme supposing that

$$x^\alpha = x_{_0}^\alpha + \xi^\alpha,$$

where ξ^α is a small additive in order of ε^2 to nonperturbed coordinates $x_{_0}^\alpha$. So, in basic approximation we get from (4) its time-like

$$x_{_0}^0 = \tau \tag{12}$$

and space-like

$$x_{_0}^k = \sum_{n=1}^{\infty} \left(\mathcal{A}_n^k \cdot \cos \frac{\pi n}{L}\tau + \mathcal{B}_n^k \cdot \sin \frac{\pi n}{L}\tau \right) \cdot \sin \frac{\pi n}{L}\rho \tag{13}$$

partial solutions.[1]

Later on we suppose that $\mathcal{B}_n^k = 0$ and $n = 1$ for simplicity. So, the basic solution have the form

$$x_{_0}^k = x_{_0}^k(\tau;\rho) = \mathcal{A}^k \cdot \cos \frac{\pi}{L}\tau \cdot \sin \frac{\pi}{L}\rho. \tag{14}$$

Then it is easily seen that time-like additive fulfils the linear homogeneous equation as before

$$\frac{d^2\xi^0}{d\tau^2} - \frac{d^2\xi^0}{d\rho^2},$$

so far the space-like additive - the nonhomogeneous one

$$\frac{d^2\xi^k}{d\tau^2} - \frac{d^2\xi^k}{d\rho^2} = \frac{\varepsilon^2}{2}\frac{dl_{_0}^k}{d\rho} l_{_0}^m l_{_0}^m = f^k(\tau;\rho). \tag{15}$$

In the following investigations it is convenient to put $\xi^0 = 0$ so that solution (12) should remain valid. Thus finally we have $x^0 = \tau$. Passing to equation (15) and having in mind that

$$l_{_0}^k = \frac{dx_{_0}^k}{d\rho} = \mathcal{A}^k \cdot \frac{\pi}{L} \cdot \cos \frac{\pi}{L}\tau \cdot \cos \frac{\pi}{L}\rho \tag{16}$$

according to (14), its right part may be converted to the form

$$f^k(\tau;\rho) = -\frac{\varepsilon^2}{8} \cdot \left(\frac{\pi}{L}\right)^4 \cdot \mathcal{A}^k \cdot \left(\mathcal{A}^m \mathcal{A}_m\right) \cdot \Phi(\tau;\rho), \tag{17}$$

where

$$\Phi(\tau;\rho) = \cos^3 \frac{\pi}{L}\tau \cdot \left(\sin 3\frac{\pi}{L}\rho + \sin \frac{\pi}{L}\rho\right). \tag{18}$$

The nonperiodical solution of (15) is the greatest interest for us. So, according to [6] we have

$$\xi^k(\tau;\rho) = -\frac{1}{192} \cdot \left(\frac{\pi}{L}\right)^3 \cdot \mathcal{A}^k \cdot \left(\mathcal{A}^m \mathcal{A}_m\right) \cdot \varepsilon^2 \cdot \tau \left(\sin 3\frac{\pi}{L}\tau \cdot \sin 3\frac{\pi}{L}\rho + 9 \sin \frac{\pi}{L}\tau \cdot \sin \frac{\pi}{L}\rho\right). \tag{19}$$

Proceeding from (19) we get the needed additive to the space-like vector

$$\breve{l}^k = \frac{d\xi^k}{d\rho} = -\frac{\varepsilon^2}{64} \cdot \left(\frac{\pi}{L}\right)^4 \cdot \mathcal{A}^k \cdot \left(\mathcal{A}^m \mathcal{A}_m\right) \cdot \tau \left(\sin 3\frac{\pi}{L}\tau \cdot \sin 3\frac{\pi}{L}\rho + 3 \sin \frac{\pi}{L}\tau \cdot \sin \frac{\pi}{L}\rho\right). \tag{20}$$

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[1] In further calculations we should regard that cosmic string performs oscillations in the form of standing waves only. This type of cosmic string oscillations was concidered in [4] firstly and was found its developing in [5], in particular.

The suitable - nonperiodical - additive to the energy-momentum tensor in accordance with (8), (16) and (20) is of the following expression

$$\breve{\Theta}^{kl} = 2\mu\Big(\underset{0}{l}^k \breve{l}^l + \delta_{kl} \underset{0}{l}^m \breve{l}_m \Big) \tag{21}$$

or in explicit form

$$\breve{\Theta}^{kl} = \breve{\mu}\varepsilon^2 \big(a^k a^l + \delta^{kl} a^m a_m \big), \tag{22}$$

where

$$\breve{\mu} = \breve{\mu}(\tau;\rho) = {}_{(1)}\breve{\mu}(\tau;\rho) + {}_{(2)}\breve{\mu}(\tau;\rho), \tag{23}$$

$$a^k = \frac{\mathcal{A}^k}{L} \tag{24}$$

and in its tern

$${}_{(1)}\breve{\mu}(\tau;\rho) = -\frac{1}{32} \cdot \mu \cdot \frac{\pi^5}{L} \cdot (a^n a_n) \cdot \tau \cdot \sin 3\frac{\pi}{L}\tau \cdot \cos \frac{\pi}{L}\tau \cdot \sin 3\frac{\pi}{L}\rho \cdot \cos \frac{\pi}{L}\rho, \tag{25}$$

$${}_{(2)}\breve{\mu}(\tau;\rho) = -\frac{3}{32} \cdot \mu \cdot \frac{\pi^5}{L} \cdot (a^n a_n) \cdot \tau \cdot \sin \frac{\pi}{L}\tau \cdot \cos \frac{\pi}{L}\tau \cdot \sin \frac{\pi}{L}\rho \cdot \cos \frac{\pi}{L}\rho. \tag{26}$$

By keeping terms from (10) having the nonperiodical character we get the Einstein equation

$$\Box \breve{h}_{kl} = -16\pi\gamma\breve{\mu}\big(a_k a_l + \delta_{kl} a^m a_m \big)\delta_3(x - x_0) \tag{27}$$

and find immediatly its solution

$$\breve{h}_{kl} = 4\gamma \int\limits_V \frac{\breve{\mu}(a_k a_l + \delta_{kl} a^m a_m)}{|x - x'|} \delta_3(x - x_0) dV' = 8\gamma\breve{\mu}\big(a_k a_l + \delta_{kl} a^m a_m \big) \ln \frac{r}{r_0}. \tag{28}$$

Now using the results of paper [3] we obtain the total metric of nonlinear cosmic string performing oscillations in the form of standing waves

$$ds^2 = \left[1 - 2\gamma\mu\varepsilon^2 \ln \frac{r}{r_0} \right] dx^{0\,2} - \left[1 + 2\gamma\mu\varepsilon^2 \ln \frac{r}{r_0} \right] dz^2 -$$

$$-\{ \delta_{kl} \left[1 - 8\gamma\mu\big(1 + \frac{\varepsilon^2}{4}\big) \ln \frac{r}{r_0} \right] - 8\gamma\breve{\mu}\varepsilon^2 \big(a_k a_l + \delta_{kl} a^m a_m \big) \ln \frac{r}{r_0} \} dx^k dx^l, \tag{29}$$

$$(k, l = 1, 2).$$

3. Gravitational radiation of nonlinear cosmic string oscillating in the form of standing waves

Our next step is the calculation of gravitational energy loss due to its radiation from nonlinear oscillating cosmic string. To calculate this energy loss we shall base on Landau-Lifshitz pseudotensor of gravitational field [7]. Let us write it down in our denotations

$$\breve{t}^{\alpha\beta} = \frac{1}{32\pi\gamma} \breve{h}^{\nu,\alpha}_{\mu} \breve{h}^{\mu,\beta}_{\nu}. \tag{30}$$

The gravitational energy flux is the mixed component of (30)

$$-\breve{t}^{0k} = \frac{1}{32\pi\gamma}\breve{h}_{nm,0}\breve{h}_{mn,k}. \tag{31}$$

Its explicit form is

$$\breve{t}^{0k} = \frac{12}{\pi} \cdot \gamma\breve{\mu} \cdot \frac{d\breve{\mu}}{d\tau} \cdot (a^m a_m)^2 \cdot \frac{1}{r} \cdot \ln\frac{r}{r_0} \cdot \eta^k = \breve{t} \cdot \eta^k \tag{32}$$

in virtue of (29).

So, the rate of energy loss through the cylindrical surface of height L is

$$-\frac{d\mathcal{E}}{dt} = \oint\limits_S \breve{t}^{0k} dS_k = \int\limits_0^L \int\limits_0^{2\pi} \breve{t} r dz d\varphi. \tag{33}$$

Substitution (32) into (33) leads to the following result

$$-\frac{d\mathcal{E}}{dt} =\approx \frac{1}{171} \cdot \gamma\mu^2 \cdot \varepsilon^4 \cdot (a^m a_m)^4 \cdot \pi^{11} \cdot \left(\frac{\tau}{L}\right)^2 \cdot \ln\frac{r_0}{r} \cdot \mathcal{F}\left(\frac{\tau}{L}\right), \tag{34}$$

where

$$\begin{gathered}\mathcal{F}\left(\frac{\tau}{L}\right) = \{3\cos 3\frac{\pi}{L}\tau \cdot \sin 3\frac{\pi}{L}\tau \cdot \cos\frac{\pi}{L}\tau - \sin^2 3\frac{\pi}{L}\tau \cdot \sin\frac{\pi}{L}\tau + \\ \frac{9}{2}\left(\cos^2\frac{\pi}{L}\tau \cdot \sin\frac{\pi}{L}\tau - \sin^3\frac{\pi}{L}\tau\right) + \frac{3}{2}\left(\cos^2\frac{\pi}{L}\tau \cdot \sin 3\frac{\pi}{L}\tau - \sin 3\frac{\pi}{L}\tau \cdot \sin^2\frac{\pi}{L}\tau\right) + \\ \frac{3}{2}\left(3\cos 3\frac{\pi}{L}\tau \cdot \cos\frac{\pi}{L}\tau \cdot \sin\frac{\pi}{L}\tau - \sin 3\frac{\pi}{L}\tau \cdot \sin^2\frac{\pi}{L}\tau\right)\}\cos\frac{\pi}{L}\tau\end{gathered} \tag{35}$$

is periodical function of time τ.

Let us make some crude estimations for the loss of gravitational energy. In the case of strong oscillations in accordance with [8] it is possible to assume

$$a^m = \frac{\mathcal{A}^m}{L} \approx 1,\ \ln\frac{r_0}{r} \sim \ln\frac{r_0}{L} \approx 10^{-2},\ \frac{\tau}{L} \approx 1. \tag{36}$$

Having in mind that $|\mathcal{F}\left(\frac{\tau}{L}\right)| \leq 10$ and $\varepsilon \geq 10^{-7}$ we finally get the standard expression for the rate of gravitation energy radiation

$$|\frac{d\mathcal{E}}{dt}| = G\gamma\mu^2, \tag{37}$$

where numerical coefficient G is in order of 10^{-25}. But it is absolutely clear that the main parameter which is determining our evaluation is the parameter of nonlinearity ε . In fact, if it tended to larder magnitudes, (if $\varepsilon \to 1$, for instance, that really took place for strings at the end of their brownian "epoch" of evolution), than G is in order of 10^3. In the case of a very strong oscillations when $a^m > 1$ [2], (during the brownian "epoch)" coefficient G can be larder then 10^3. We note in passing that for linear cosmic strings G is in order of 10^2 as it was argued in [8].

In other words, the gravitational radiation of nonlinear cosmic strings was very strong at the earliest stages of Universe evolution, it had grown quadratically in time (see expression(34)) up to the period when expansion had depressed nonlinear oscillations and reduced them to the linear ones with the well-known regime of emission [9].

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[2] In general it is impossible to use gravitational energy flux type of (32) in such extremal situation. But our estimations have rather crude character, as it was pointed out later.

4. Results and discussion

This work develops ideas of paper [3] where the energy-momentum tensor of nonlinear cosmic string, the metric of nonlinear cosmic string and its equation of motion were found for the first time.

Now it was considered the gravitational energy radiation from nonlinear cosmic string or cosmic string performing the nonlinear oscillations. It was shown that metric becomes nonstationary because its components depend on time linearly. But the rate of gravitational energy radiation depends quadratically on time. Thus the gravitation energy emission from cosmic strings was stronger at the earliest stages of Universe evolution than it was been argued in the previous papers.

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