Using Euler Buckling Springs for Vibration Isolation.

J. Winterflood, T. Barber, D.G. Blair

Department of Physics, University of Western Australia, Perth 6009, Australia.

Abstract
Difficulties in obtaining ideal vertical vibration isolation with mechanical springs is identified as being due to the mass of the elastic element which is in turn due to its energy storage requirement. A new technique to minimise this energy is presented - being an Euler column undergoing elastic buckling. The design of a high performance vertical vibration isolation stage based on this technique is presented together with its measured performance.

1 Introduction

The test masses in an interferometric gravitational wave detector require isolation from seismic motion to an unprecedented degree and these stringent demands have provided the motivation for the development of greatly improved vibration isolators [1]. We present a new use of elastic springs which allows vertical vibration isolation to achieve performance comparable to the horizontal vibration isolation of a simple pendulum.

Vertical vibration isolation presents more problems than horizontal isolation because of the strong gravitational force that must be counteracted. Vibrational motion (between the support and the isolated mass) in the presence of gravity requires the dynamic storage of significant amounts of energy (*mgh*) to momentarily absorb this motion. This energy storage is often provided in the form of mechanical springs such as the coil spring suspension shown in fig 1. In order for a *linear* spring (ie displacement $\propto$ force) such as this to achieve the same resonant frequency as a pendulum ($\omega=(g/l)^{1/2}$), it must be stretched under load (beyond its relaxed length) by the same length (*l* in fig 1) as the equivalent pendulum.

Most high performance vertical vibration isolators operate in a regime where the amplitude of vibration is very small compared with their extension under load. Under vibration only this small amount of dynamic energy is exchanged in and out of the spring while a large amount of energy remains statically in the spring due to its initial extension under load. The key point here is that the total energy stored in the spring is much larger than the dynamic energy storage required in operation, as fig 4 illustrates.

The large static energy storage necessitates a large mass of elastic material - proportional to the total static and dynamic energy to be stored. (In a pendulum on the other hand there is in principle no static energy and the dynamic energy is stored as gravitational potential energy - not requiring elastic material with mass.) The disadvantage of the large mass requirement is that it supports low frequency *internal* resonant modes of the spring (eg "surging" in coil springs). These resonances have a large effective mass which strongly couple vibration

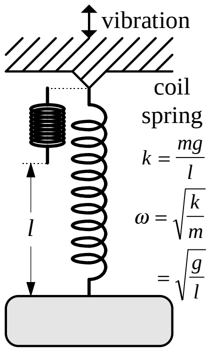


Fig 1

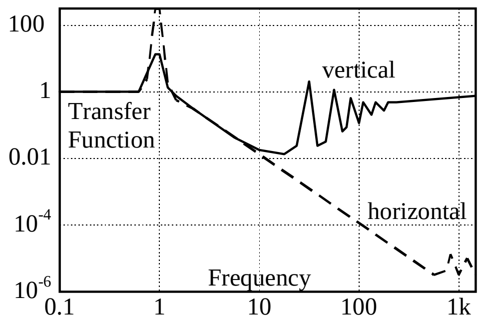


Fig 2

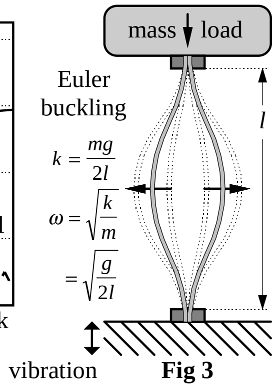


Fig 3

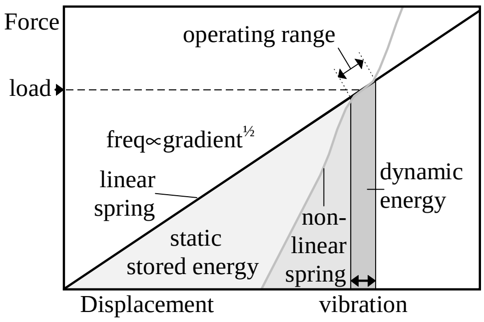
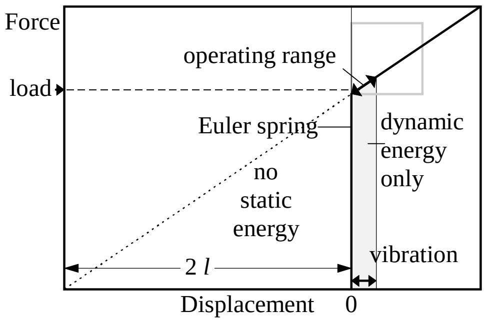


Fig 4



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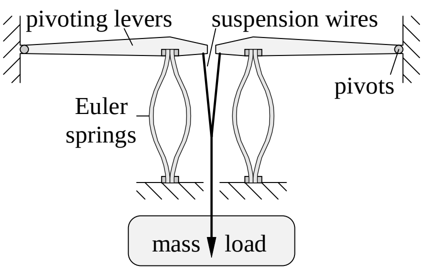

Fig 6

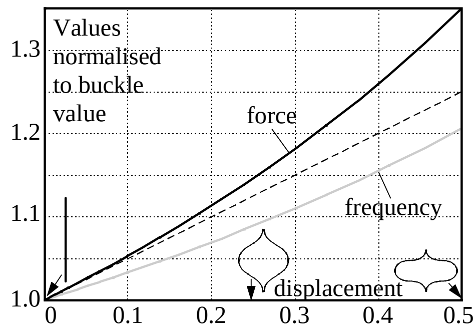

Fig 7

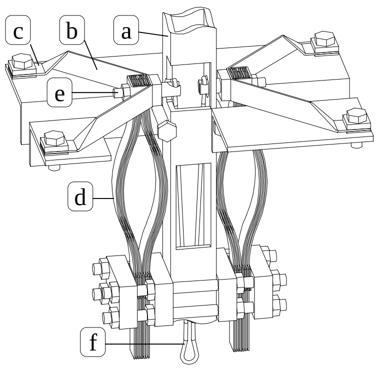

Fig 8

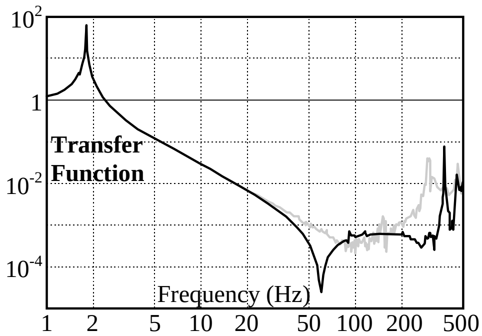

Fig 9

between the support and the isolated mass at these internal resonant frequencies. After providing text book performance at low frequencies as illustrated by the solid curve in fig 2, internal resonances appear (about 30Hz in fig 2) and increase in mode density so that performance at higher frequencies is strongly degraded.

The same effect also occurs in the pendulum, but because the suspension fibre can be made so much less massive than the spring, its first internal (violin string mode) resonant frequency is much higher. In addition it couples much less energy due to the large ratio of the suspended mass to the fibre mass (the coupling depends also on its Q-factor).

2 Advanced Vertical Isolation

There are three main areas to consider in order to alleviate this spring mass problem inherent in mechanically sprung vertical vibration isolators:- (1) ensure that the entire volume or mass of spring material is usefully storing energy by being stressed to its limit, (2) redistribute the mass of the spring to minimise its velocity and thus the kinetic energy of any internal mode motion, and (3) reduce the static energy and mass of the spring while keeping a low resonant frequency by producing a non-linear force vs displacement relationship.

1) The first of these areas only offers a small gain (a maximum of 2 for torsion and 3 for bending) and is rarely considered. A simple example would be to wind a coil spring from a tube rather than from solid material. This removes the central mass which is scarcely stressed and does not store much energy. This is difficult to apply to bending. However when a spring is only stressed in one polarity, then in principle it is possible to preset initial stresses within the material so that the entire volume is stressed to its limit at some maximum deflection. For material loaded in torsion this is commonly called “scragging” and consists in plastically twisting the material well beyond its initial yield point in the direction of later loading. After release, the surface layers have negative strain, and deeper layers retain positive strain which is prevented from relaxing by the surface layers. This spring can now be elastically twisted in the scragged direction considerably further than before the treatment, and since the spring-rate is unaltered, the uni-directional energy storage capability has been improved significantly. The equivalent approach for flat springs is similar - start with an appropriate shape to be able to bend it well past yield in the loaded direction before obtaining the desired operating shape. An example of this is curved cantilever blades and we believe this technique also results in reduced creep at high stress levels[2] as any tendency or weakness to creep has been forced to occur during plastic forming.

2) Some examples of improved distribution of spring mass are well known. A cantilever blade[3] with constant stress over its surfaces is in the shape of a triangle (top view) with the fixed attachment being the wide base of the triangle and the suspending tip being the apex. This puts most of the mass near the fixed attachment where motion is minimum thus achieving higher internal mode frequencies. Probably the best that can be achieved is the torsion rod suspension[4]. Here the crank-arm can be made very rigid (to have very high internal mode resonances) and the spring material is all located very close to the fixed centre of rotation.

3) Several examples of reducing the static energy and mass stored in the spring using non-linear spring techniques have appeared in the literature recently. One method adds powerful magnets to a conventional linear spring which strongly repel and try to drive the mass away from the normal operating position[5]. This is an anti-restoring force or “anti-spring” which when added to the springs normal restoring force produces a region of reduced gradient on the force displacement curve as is illustrated by the grey curve in fig. 4. The main resonant frequency of an isolation system is determined by the (square root of the) spring-rate which is the gradient of the force-displacement characteristic. By operating in the flattened region of

the curve, a much lower resonant frequency is obtained than would normally be the case for the static displacement and energy storage involved. Alternatively the geometry of the structure can be arranged to produce a non-linear spring. The torsion-crank suspension is an example of this technique [6].

Since the mass of spring material used must be proportional to the total (static + dynamic) energy stored in the spring, it is clear from figure 4 that for a given resonant frequency, the spring mass used in these types of non-linear systems can be greatly reduced from its linear equivalent. It also becomes apparent that the best non-linear spring would be one where the static energy could be reduced to zero so that only dynamic energy storage need contribute to the spring mass. Remarkably this ideal is readily obtained by the very simple spring arrangement described hereafter.

3 Euler Buckling Spring

It is well known in engineering that a column of elastic material will support a load with virtually no deflection until at some critical value of load (dependent on its modulus and not on its yield strength) it suddenly starts to buckle. This is the sort of wall-shaped non-linearity (fig 5) that is required in the force-displacement characteristic to provide zero static energy. The shape assumed by an inextensional elastically buckling spring is called an *elastica* and exact analysis of large deflections in such a spring involves the use of elliptic integrals [7]. The theoretical force-displacement characteristic of such a spring under elastic buckling can be readily calculated and is shown as the black curve in fig 7. The displacement is normalised to the flexure length, and the force is normalised to the critical buckling value for a column with fixed ends $P_{cr}=4\pi^2EI/l^2$ where E is modulus of elasticity, I is the area moment of inertia and l is the length of the column. The black curve shows that the force vs displacement characteristic is remarkably well behaved with a low spring-rate which remains reasonably constant even up to very large amounts of buckling. It should be noted that the displacement axis in fig 7 covers a very large travel range, compressing the Euler springs to half their initial length (see spring shapes indicated), whereas a useful displacement (for minimising static energy etc) would typically compress by less than 1% of spring length. It is apparent that in theory the spring rate remains almost constant for such small displacements.

A simple expression $k=mg/(2l)$ is obtainable for the initial spring-rate (at start of buckling) and is indicated by the dashed line in fig 7. Figure 5 shows this initial spring-rate slope (now in black) in the context of the critical load, while the grey rectangle indicates the area expanded in fig 7. Remarkably the vertical resonant frequency of the Euler mass-spring system $\omega_e=(g/(2l))^{½}$ obtained depends only on the length of the spring (and g = accel due to gravity). This is strongly reminiscent of the analogous expressions for the linear coil-spring system $\omega=(g/l)^{½}$ where l is instead the extension of the linear spring under load (fig 1). However for the Euler spring an extra factor of 2 is involved - so that the suspended mass moves as though it was suspended by a linear spring which had been extended by an amount *twice* the length of the unbuckled Euler spring. Figure 7 also shows how the resonant frequency varies with the degree of buckling relative to its value at the start of buckling. It can be seen that it only increases by 20% for a 50% compression of spring length.

Figure 5 illustrates that if the working range is designed to start just at the onset of buckling, then all of the energy stored by this type of spring is the dynamic energy exchanged in and out while operating within its working range. It also becomes apparent that the mass of spring required is directly proportional to the working range that can be accepted. For example suppose we can accept a working range of ½mm at a resonant frequency of 1Hz (normally requiring 25cm static extension in a linear system), then if made of the same elastic material the Euler spring may be $1/250^{th}$ the mass of its linear equivalent. One would expect the internal resonant modes of the Euler spring to be roughly proportional to the square root of this mass ratio (ie $\sqrt{250} \approx 16$ times higher than the linear case!) and the effect of the coupling at the internal resonances to be reduced by this mass ratio (ie $1/250^{th}$). In addition various methods of spring-rate reduction may be applied, to allow, in principle, even greater relative improvement than this.

4 Supporting Structures

In order to be useful as a suspension device the Euler springs need to be constrained within some structure to limit motion to the desired longitudinal compression. A simple mechanism to provide this constraint is a cantilever or pivoted lever arm. A pair of these levers operating in a balanced manner is shown in figure 6. The motion constraint in this case is the arc of a circle rather than a straight line, but it is approximately linear for the small working range required.

The slightly non-linear motion also provides some advantages. One is that the load can be supported or suspended at a different distance along the lever to that at which the spring blade is clamped - allowing a mechanical advantage ratio to be used to match an available spring to a particular load. This lever ratio may be adjustable to allow a continuous trade-off between supported mass and working range (mass × range = constant fixed by spring blade energy storage capacity). Also the rotational motion allows various spring-rate reduction techniques to be applied. Some of these are more fully described in the papers [8,9].

5 A High Performance Vertical Vibration Isolator

A single stage vertical isolator based on fig 6 was designed and built into the form shown in fig 8. This particular implementation was designed to be hung in a cascaded chain of stages one below the other. The square tube (a) at the centre is the mounting frame for all the parts comprising the isolation stage (only the suspension components are shown and sections are cut away for viewing). The pivoting arms (b) are a wishbone shape for maximum rigidity and minimum mass. They are made of sheet metal and are intended to flex in the flat sections (c) near the clamp which can be thinned for this purpose. The Euler springs (d) are made of strips of feeler gauge stock, clamped together with spacers (of the same material) between each blade. The special bolts (e) clamping the upper ends of the blades pass through large clearance holes in the central tube to limit their motion to a safe operating range and contain a conical socket to engage firmly with the folded over ends of a loop of suspension wire (f). The suspended mass or following stages are hung by a pin through the centre of this loop of suspension wire.

The transfer function in fig 9 was for the structure of fig 8 but with a total of only two Euler spring blades (one each side bending outwards). Having blades only bending in the outward direction is normally unstable but can be stabilised by the contributing stiffness of the flexing wishbones (fig 8c) and bending wire (f). The spring blades were 0.8mm thick strips of feeler gauge with a length between clamps of 126mm and the system was loaded with a mass of 32kgs. It was designed to have a working range of ±1mm (ie 2mm total from unbuckled to motion limit). For a shake-test measurement, the structure was inverted so that the mass of the structure together with extra mass clamped to the tube (a) is all suspended by the springs (d) and wire loop (f). The wire was shaken vertically with a swept sine and the ratio of the wire motion to the suspended mass motion measured with vertical geophones. Figure 9 shows that the first troublesome internal mode occurs around 400Hz. Evidence mentioned below indicates that this mode is the mass of the clamping bolts (e) resonating with the wire (f) and that the first blade internal mode is the small one appearing at almost 500Hz.

There is a clear notch at 60Hz and above that an isolation floor of -65 to -70dB. This is strongly suggestive of dynamic inertia (or centre of percussion) effects. To test this we loaded the wishbones (b) at about their mid point with small blocks of lead (of order 60g each side) with the resulting transfer function shown in grey. The notch at 60Hz disappeared as expected and a rounded shape falling to -70db around 100Hz was obtained confirming this suggestion. In addition the first internal mode resonance moved from 400Hz down to 300Hz indicating its dependence on the wishbone and clamping bolt mass, while the small mode near 500Hz remained unaltered. This indicated that the 400Hz mode was due to the wire and clamping bolts, rather than the Euler spring blades. Normally with very flexible joints at (c), this internal mode would not couple significantly, but in this particular case we had deliberately made the joints (c) very stiff to stabilise the negative spring-rate of the outward bending blades. The reason we were particularly interested in this configuration was because with the design of fig 8 it allows the outward bending blades (f) to be clamped very close to the tube (a) and wire (f), minimising forces in the wishbone (b).

6 Conclusion

Vertical isolation using mechanical springs (as opposed to compressed gas or magnetic suspension) has been plagued by the problem of internal modes bypassing the isolation at relatively low frequencies. The new technique presented here, which is most applicable for suspending constant loads under conditions of very small vibration, is capable of providing orders of magnitude better performance than previous approaches.

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