LISA and gravitational wave stochastic backgrounds

Alberto Vecchio †
School of Physics and Astronomy, The University of Birmingham, Edgbaston,
Birmingham, B15 2TT, United Kingdom

Abstract. A variety of gravitational wave stochastic backgrounds populate the sensitivity window of LISA: signals produced by galactic and extra-galactic binary systems, and relic gravitons generated in the early-Universe. We review our present astrophysical understanding of the main sources, address the prospects of detection with LISA, and discuss possible follow-on missions.

1. Gravitational wave stochastic backgrounds

Gravitational wave experiments will play a key role in the investigation of the frontiers of cosmology and fundamental physics by observing the stochastic background produced in the early-Universe. In fact, the Universe became "thin" to gravitational waves (GWs) at the Plank epoch, corresponding to the cosmic time $\sim 10^{-43}$ sec.; the gravitons decoupled from the surrounding plasma at a temperature of the order of the Planck mass $\sim 10^{19}$ GeV, and gravitational radiation produced at that epoch or later, such as the electro-weak and the grand unification scale, has traveled undisturbed to us, carrying full information about the state of the Universe, and the physical processes from which it took origin (see [1] and references therein).

Several classes of astrophysical sources will also be studied through the GW stochastic background that they generate. They include short-period binary systems – such as close white dwarf binaries (CWDBs), W UMa binaries, and neutron star/solar mass black hole binaries [2, 3]– and massive black hole binary systems characterized by an extreme mass ratio ($\sim 10^{-5}$) [4] in the mHz range, rapidly rotating neutron stars below ≈ 50 Hz, and supernovae, in the kHz band. Most of the mentioned sources are too faint to be detected individually, but the incoherent superposition of GWs from the whole population (up to cosmological red-shift) is recorded as a stochastic signal at the instrument output, the so-called astrophysically generated GW stochastic background. The observation of stochastic backgrounds will therefore have a profound impact on astronomy.

Over the next decade a very wide band of the GW spectrum will be fully accessible. Ground-based laser interferometers (LIGO [5], GEO600 [6], VIRGO [7] and TAMA [8]) operating in the frequency region $\sim 10\,\mathrm{Hz} - 1\,\mathrm{kHz}$, will achieve an initial sensitivity

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† E-mail: av@star.sr.bham.ac.uk

$h_{100}^2\,\Omega_{\rm gw} \sim 10^{-6}$, which will steadily improve and reach $h_{100}^2\,\Omega_{\rm gw} \sim 10^{-10}$ when the detectors operate with "third generation" technology [9]. In space, ESA and NASA will launch in 2011 the Laser Interferometer Space Antenna (LISA), a space-borne laser interferometer aimed at the low-frequency window $\sim 10^{-5}\,{\rm Hz} - 10^{-1}\,{\rm Hz}$ [10]. The LISA design sensitivity is such that signals with $h_{100}^2\,\Omega_{\rm gw}(f) \sim 10^{-10}$ will completely dominate the detector output over the entire observational band. In addition to laser interferometry, other techniques could play an important role in searching for stochastic backgrounds: several refined resonant detectors are presently in planning and proposal stage, and could achieve an interesting sensitivity in the frequency range $1\,{\rm kHz} \lesssim f \lesssim 10\,{\rm kHz}$ [11]; and a prototype detector is now working in the very high frequency band $f \sim 100$ MHz, which might lead to the development of a novel instrument aimed at GW cosmology in this new frequency regime [12].

Among the instruments under construction, LISA could well represent the most suitable device to search for stochastic backgrounds. Several cosmological models predict a primordial background with a flat energy density spectrum [1], whose characteristic amplitude therefore increases at low frequencies: $h_c(f,\Delta f) \simeq 7.1 \times 10^{-22}[h_{100}^2\,\Omega_{\rm gw}(f)/10^{-8}]^{1/2}\,(f/1\,{\rm mHz})^{-3/2}\,(\Delta f/3.2{\times}10^{-8}\,{\rm Hz})^{1/2}$. Backgrounds generated by a variety of binary systems both galactic [2, 13, 14] and extra-galactic [3, 4, 15] are *guaranteed* signals for LISA, with several components that dominate the detector output over a wide frequency range. Recently stochastic backgrounds have been at the centre of intense studies, and several new results are changing our view of the science impact of LISA. They are also stimulating a broader discussion regarding follow-on missions, possibly with GW cosmology as primary goal. The picture that is emerging can be summarized as follows‡:

- If the primordial background is at the level $h_{100}^2\,\Omega_{\rm gw}(f) \sim 10^{-10}$, then it should be easily detectable by combining the Sagnac and Michelson observable that can be constructed from the read-outs of the three arms of LISA [16, 17]; in this case, however, generated backgrounds are likely undetectable, unless their spectral content largely exceeds the current theoretical estimates; over long integration times ($T_{\rm obs} \approx 3$ yrs) one should still be able to pull out information on the background from galactic CWDBs, by exploiting the characteristic periodic modulation of the auto-correlation function of the data which is produced by the anisotropy of this signal in the Michelson observable [18].
- If the spectrum of the primordial background is in the range $10^{-11} \lesssim h_{100}^2\,\Omega_{\rm gw}(f) \lesssim 10^{-10}$, then the primordial background should still be detectable in some portion of the LISA sensitivity window, possibly around 0.1 mHz; the isotropic component of the signal generated by galactic CWDBs will be detectable using the Michelson/Sagnac observable, mainly at $f \sim 1$ mHz, and the study of the anisotropy

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‡ We will refer to "primordial" and "generated" backgrounds to indicate GW stochastic signals generated, respectively, in the early-Universe (e.g. relic gravitons emitted during inflation), and by the incoherent superposition of GWs from a very large number of unresolved astrophysical sources (e.g. white-dwarf binary systems).

of galactic contributions will be carried out at higher signal-to-noise ratio (SNR). Our present theoretical understanding suggests that the chance of detecting extra-galactic backgrounds from solar mass sources [3] will be rather bleak [18].

- If the primordial background is well below $h_{100}^2\,\Omega_{\rm gw}(f) \sim 10^{-11}$, then this signal is not observable by LISA. Actually, if $h_{100}^2\,\Omega_{\rm gw}(f) \lesssim 10^{-12}$ even the cross-correlation of two independent LISA detectors will not be able to detect the primordial component, because of the presence of generated backgrounds [19]. The LISA frequency window would be therefore fully open to thorough studies at SNR $\gtrsim 10$ of generated backgrounds, both from galactic and extra-galactic sources.
- In order to achieve a better sensitivity one needs to cross-correlate the data from two instruments with uncorrelated noise. If the detection of the background from the early-Universe is the main science goal, then the detector sensitivity has to be tuned on a frequency region free from generated backgrounds (which is clearly not the mHz range), as they represent a fundamental sensitivity limit [19]. In this respect, a promising frequency window seems to be ~ 0.1 Hz -1 Hz; possible follow-on missions would therefore have shorter (by a factor ~ 100) arm-length, higher laser power, larger "optics", and more stringent requirements on the accuracy of the phase measurement.

The next sections are devoted to a more detailed discussion of some of the issues described above, and provide pointers to future work.

2. Detecting stochastic backgrounds

The key issue in searching for stochastic backgrounds with LISA is to identify unambiguously the GW signal in one single data stream. In fact, a stochastic background is a random process which is intrinsically indistinguishable from the detector noise. On ground, this problem is easily overcome by cross-correlating the outputs of two instruments separated by a distance smaller than half of the typical wavelength of the signal $\lambda_{\rm gw} \simeq 6000\,(f/50\,{\rm Hz})^{-1}$ km [20] (as an example, the two LIGO sites, Hanford and Livingston, are $\simeq 2998$ km apart): being the signal in the two detectors "the same", and the noise (in principle) uncorrelated, the SNR increases as $\sqrt{T}_{\rm obs}$ [9]. In space we do not have (at present) the luxury of two instruments, and it has been wondered to what extent LISA could contribute to improve our understanding of stochastic backgrounds.

Recently, a series of papers have pointed out that one can recombine the read-outs from each arm of LISA in such a way to compute the so-called "symmetrized Sagnac observable" (see [16] and references therein). In fact LISA *works as* a Michelson interferometer when the read-outs on board of each spacecraft are combined to obtain the phase difference in the round-trip laser light along two adjacent arms. Several other combinations of the laser signals are possible. One of these, the symmetrized Sagnac observable, is produced by taking the phase difference of the laser light sent clockwise and counter-clockwise around the LISA constellation, and averaging over the outputs from the three vertices. The remarkable property of the symmetrized Sagnac observable is to be almost insensitive to GWs at frequencies smaller than the inverse of the light travel time between two corners of the instrument, $f \simeq 10^{-2}$ Hz. In practice, one can construct (in software) a GW shield, and therefore a mean of calibrating the instrument. By combining the symmetrized Sagnac observable with the Michelson observable, which is sensitive to the signal and the noise, one can therefore detect with confidence a GW stochastic background. It has been recently argued that this techniques might be able to achieve a sensitivity close to the one provided by the cross-correlation of two LISA detectors [17], which is of the order $h_{100}^2\,\Omega_{\rm gw} \sim 10^{-13} - 10^{-12}$ for an integration time of 4 months, depending on the location and relative orientation of the instruments [19]. A careful analysis of the noise sources that could degrade the sensitivity of the instrument using the symmetrized Sagnac observable (and the actual technical implementation of the scheme of reading-out and recombining the laser signals) is now needed, but this recent work has definitely shown that LISA will be effective in searching for stochastic backgrounds.

3. Studying the anisotropy of the stochastic background

Many signals detectable by LISA carry a strong signature of anisotropy: for example galactic sources, such as CWDBs, are mainly distributed in the disk, and LISA is placed in a peripheral location of the Galaxy. Even the primordial background, which is *intrinsically isotropic* to a high degree, is characterized by a dipole component in the LISA data set due to the proper motion of the Galaxy with respect to the cosmological rest frame.

The key idea to detect an anisotropic GW stochastic background is to exploit the periodic change of the LISA orientation – the instrument barycentre follows an heliocentric orbit with period $T = 1$ yr, and the detector plane, tilted by 60° with respect to the Ecliptic, counter-rotates with the same period T [10] – which produces modulations of the auto-correlation function of the data (LISA has a quadrupole antenna pattern). This idea was firstly suggested in [21], and has been thoroughly explored in [18]. The data analysis strategy is simple, and relies only on the Michelson observable. The data set of length $T_{\rm obs}$ is divided into several chunks of much smaller size $\tau \ll T < T_{\rm obs}$ (e.g. $\tau \sim 10^5$ sec and $T_{\rm obs} \sim 10^8$ sec), over which the detector location and orientation are essentially constant. One constructs the new signal

$$S(t) = \int_{-\infty}^{+\infty} df\, \tilde{o}(f,t)\, \tilde{Q}(f)\, \tilde{o}^*(f,t)\,, \tag{1}$$

where $\tilde{o}(f;t)$ is the Fourier transform of the data segment of length τ, centered around t, and $\tilde{Q}(f)$ is a suitable filter function that can be determined by maximizing the SNR [9]. One then searches for peaks in $S(t)$ at multiples of the LISA rotation frequency $1/T$: because $\tau \ll T$ the correlation varies as LISA changes orientation. Since the LISA motion is periodic, one can decompose the mean value of $S(t)$ as a Fourier series $\langle S(t)\rangle = \sum_{-\infty}^{+\infty} e^{i\,2\pi m t/T}\langle S_m\rangle$. The amplitude of each harmonics S_m represents the observable, which can be detected at a signal-to-noise ratio

$$\left(\frac{\mathrm{S}}{\mathrm{N}}\right)_m = T_{\mathrm{obs}}^{1/2}\,\Theta_m^{(0)}\left\{\int_0^\infty df\,\mathcal{J}^0(f)\right\}^{1/2}, \tag{2}$$

where

$$\mathcal{J}^{(0)}(f) \equiv \frac{4\,S_h(f)^2}{4\,\gamma_0^2\,S_h(f)^2 + 20\,\gamma_0\,S_h(f)\,S_n(f) + 25\,S_n(f)^2}, \tag{3}$$

and

$$\Theta_m^{(0)} = \left|\sum_{l=|m|}^{\infty} P_{lm}\gamma_{lm}^{(0)}\right|. \tag{4}$$

In Eqs. (2), (3), and (4) $\gamma_0 = 3/4$ is the overlap reduction function [9, 20] for two co-located and co-aligned detectors, $\gamma_{lm}^{(0)}$ are the generalized overlap reduction functions corresponding to the l-th multipole moment and the m-th harmonic, P_{lm} are the multiple moments that describe the angular distribution of the background and $S_n(f)$ is the LISA noise power spectral density (see [18] for more details). Two are they key features of LISA in observations of anisotropic stochastic backgrounds: (i) LISA is sensitive only to quadrupole ($l = 2$) and octupole ($l = 4$) anisotropy§, and (ii) the LISA sensitivity limit, that is the SNR that can be achieved for $h_{\mathrm{c}} > h_{\mathrm{rms}}$ (where h_{c} is the characteristic amplitude of the background and h_{rms} the r.m.s. amplitude of the noise) is

$$\left(\frac{\mathrm{S}}{\mathrm{N}}\right)_m \approx 4.2\times 10^2\,\Theta_m^{(0)}\left[\left(\frac{\Delta f}{10^{-3}\,\mathrm{Hz}}\right)\left(\frac{T_{\mathrm{obs}}}{10^8\,\mathrm{sec}}\right)\right]^{1/2}. \tag{5}$$

In fact it is straightforward to show that $\gamma_{lm}^{(0)} = 0$ for l odd and $l > 4$, and $\mathcal{J}^{(0)}(f)$ takes the asymptotic form $\mathcal{J}^{(0)}(f) \simeq 1/\gamma_0^2$ for $h_{\mathrm{c}} \gg h_{\mathrm{rms}}$ and $\mathcal{J}^{(0)}(f) \simeq [2S_h(f)/5S_n(f)]^2$ in the opposite limit.

Despite the theoretical uncertainties surrounding GW sources, a fair amount of information is already available concerning the distribution of binary systems in our Galaxy. Through Eqs (2)-(4) and/or Eq (5) it is easy to estimate the SNR that can be achieved. Here we assume $T_{\mathrm{obs}} = 10^8$ sec and $\Delta f = 10^{-3}$ Hz centered around $f \sim 10^{-3}$ Hz. Using Eq. (2) or (5) one can re-scale the results for different observation times and frequency bands. For sake of simplicity we also assume that the stochastic background dominates the instrumental noise (or any other competing stochastic signal), which is likely true for CWDBs, but not known for other potential sources. If $h_{\mathrm{c}} \lesssim h_{\mathrm{rms}}$, the SNRs presented in the following are reduced by $\approx 0.3\,[h_{\mathrm{c}}(f)/h_{\mathrm{rms}}(f)]^2$. CWDBs are mainly located in the Galactic disk (with typical height and radius of 0.3 kpc and 5.5 kpc, respectively) [2]. In this case the SNR can be fairly high: $(\mathrm{S/N})_1 \approx 20$ and $(\mathrm{S/N})_2 \approx 60$.

§ Notice that this is strictly true only if the LISA transfer function does not depend on the frequency; here we are considering the response of the detector in the long wavelength approximation, in which case the transfer function is indeed constant. For $f \gtrsim 10$ mHz the wavelength becomes comparable or smaller than the distance between two corners of the LISA constellation, and the transfer function is frequency dependent; in principle LISA becomes sensitive to the infinite set of multiple moments. For more details see [18, 23, 24]

Other potential GW sources could be characterized by a spherical distribution. For example, it has been suggested that MACHOs in the galactic halo could be made of low-mass binary black holes [22]; assuming that the characteristic radius of the distribution is r_c, with $r_c \gtrsim r_{\rm GC}$ – $r_{\rm GC} \simeq 8.5$ kpc is the distance of the solar system from the Galactic Centre – one obtains $({\rm S/N})_1 \approx 17\,(8.5\,{\rm kpc}/r_c)^2$ and $({\rm S/N})_2 \approx 10\,(8.5\,{\rm kpc}/r_c)^2$. Some presently unknown population could reside in the galactic bulge ($r_c < r_{\rm GC}$); for a fiducial characteristic radius $r_c = 1$ kpc, we have: $({\rm S/N})_1 \approx 132$ and $({\rm S/N})_2 \approx 13$. The former results suggest that LISA will offer a radically new mean of exploring compact objects in our Galaxy, and studying their distribution, possibly disentangling different populations. It is clear that the task of pin-pointing the exact structure of the the population(s) will be highly non-trivial. A strategy for constructing GW sky maps, and tackling the inverse problem has been suggested in [23]; however, the moderate sensitivity of the instrument will allow to extract only broad features, and not high resolution maps. The former results also yield that it is unlikely to be able to gain significant information on the much smaller degree of anisotropy produced by extra-galactic generated backgrounds, for which $\Theta_m^{(0)} \ll 1$, even if $h_c \gg h_{\rm rms}$, see Eq. (5).

It is worth considering whether LISA would stand any chance of detecting the induced dipole anisotropy of the primordial background due to the motion with $v/c \sim 10^{-3}$ of our local frame with respect to the Hubble flow. Unfortunately, the first "visible" multipole moment is $l = 2$, and not $l = 1$. This implies that $\Theta_m^{(0)} \lesssim 10^{-6}$, which essentially rules out any possibility of detecting this signature.

4. Follow-on missions

The observation of the stochastic background produced in the early-Universe represents one of the main goals of GW experiments. Recent work [19] has shown that one of the most serious factors that might prevent us from observing the primordial background is the stochastic radiation produced by a plethora of unresolved GW sources and not the instrument noise. The key issue here is the choice of the frequency band. Two competing effects need to be considered. The incoherent superposition of GWs from a reach population is recorded as a stochastic signal at the detector output when the number of sources per frequency bin, $dN(f)/df$, is > 1; in general, the lower the frequency the larger $dN(f)/df$ (for binary systems, $dN(f)/df \propto f^{-11/3}\,\mathcal{R}\,\mathcal{M}^{-5/3}\,T_{\rm obs}^{-1}$, where $\mathcal{R}$ and $\mathcal{M}$ are the merger rate and chirp mass, respectively). As a consequence, the higher the frequency the "cleaner" the observational window. On the other side, if we assume that the primordial background is characterized by a constant spectrum, and not sharply peaked at a particular frequency, then the lower the frequency the stronger the signal: $h_c(f) \propto {\Omega_{\rm gw}}^{1/2}\,f^{-3/2}$. Based on the latter criterion one would opt for a low frequency detector.

To get some insight into possible future developments, it is useful to compare the expected sensitivity of proposed instruments with the available theoretical predictions: slow-roll inflation predicts $h_{100}^2\,\Omega_{\rm gw} \sim 10^{-16} - 10^{-15}$ [25]; the cross-correlation of the data streams from "third generation" ground-based interferometers will likely yield a sensitivity $h_{100}^2\,\Omega_{\rm gw} \sim 10^{-11} - 10^{-10}$ [9]; if two identical LISA detectors were to fly, cross-correlation experiments in the mHz range (the optimal sensitivity band for LISA) would achieve $h_{100}^2\,\Omega_{\rm gw} \sim 10^{-13}$ [19]. Technology clearly points toward the low-frequency regime to achieve very high sensitivity, but the window accessible by LISA is swamped by unresolved binary systems. A follow-on mission aimed at GW cosmology will definitely need two instruments tuned on a different frequency range. The window from $\sim 10^{-6}$ Hz to $\sim 10^{-1}$ Hz seems to be severely corrupted by backgrounds produced by a number of sources: massive back hole binary systems [26], low-mass black holes orbiting massive black holes [4], white-dwarf binaries, neutron star and solar mass black hole binaries [2, 3]. Our present theoretical understanding and observational evidences of possible GW sources strongly suggest that the most suitable frequency region is between 0.1 Hz and 1 Hz, right in the middle of the sensitivity window of the planned experiments. In order to achieve the sensitivity to test slow-roll inflation one needs to design a space-based interferometer capable of reaching $h_{\rm rms} \sim 10^{-24}$ [19], which represents a very considerable challenge: stringent requirements have to be imposed on the power and frequency of the laser as well as on the dimension of the optics. Shot noise, beam pointing fluctuations, and accuracy of the phase measurement are the most likely sources of noise in this band. However, the observational band will still be contaminated by several (quite weak) signals, that need to be carefully removed before searching for the primordial background. How accurately this cleaning can be done, and what technique(s) is suitable are open issues that deserve careful scrutiny.

Gravitational wave experiments are not the only ones able to detect the GW background from the early-Universe. Cosmic microwave background (CMB) experiments, such as PLANCK [27], might be capable, although *indirectly*, of setting upper-limits on, or even detecting such signal by the end of the decade. This can be achieved by studying the CMB polarization, and searching for the signature induced by the GW background [28]. These new data, together with Earth-based GW observations and the technology developments toward the LISA mission (including SMART-2) will provide key information to undertake the challenge of *directly* studying the Universe in its very first second of life.

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