The Cassini Ka-Band Gravitational Wave Experiments

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Abstract.

Cassini, a joint American/European interplanetary scientific mission to Saturn, will be continuously and coherently tracked for forty days during its solar oppositions in the next three years, starting on November 26, 2001. Doppler tracking searches for gravitational waves in the millihertz frequency band will be performed by using newly implemented Ka-Band ($\approx$ 32 GHz) microwave capabilities on the ground and onboard the spacecraft. Use of the Ka-Band coherent microwave link will suppress solar plasma scintillations to levels below those identified by remaining instrumental noise sources, making the Cassini Doppler tracking experiments the most sensitive searches for gravitational waves ever attempted in the millihertz frequency band.

This paper provides a short review of the Doppler response to gravitational radiation, the noise sources and their transfer functions into the Doppler observable, and estimates of the anticipated Cassini Doppler tracking sensitivities to gravitational radiation.

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1. Introduction

Doppler tracking of interplanetary spacecraft is a technique for searching for gravitational radiation in the millihertz frequency band. The Earth and the spacecraft act as free test masses, and the Doppler tracking system continuously measures their relative dimensionless velocity, $\Delta v/c = \Delta\nu/\nu_0$ (here Δv is the relative velocity, $\Delta\nu$ is the Doppler frequency fluctuation, and ν_0 is the nominal frequency of the microwave link). A gravitational wave of strain amplitude $h(t)$ propagating through the radio link causes small perturbations in the Doppler time series of $\Delta\nu(t)/\nu_0$. These perturbations are of order h and are replicated three times in the tracking record with a characteristic pattern that depends on Earth-spacecraft distance and the source-Earth-spacecraft angle [1]. These three events in the time series can be thought of as due to the gravitational wave (GW) buffeting the Earth, then the spacecraft, and the original Earth buffeting event transponded back to the Earth at a time $T_2 \equiv 2L/c$ later, where L is the distance between the Earth and the spacecraft, and c is the speed of light. The sum of these three Doppler perturbations is zero. This overlap of the three events, and partial cancellation, occurs for GW pulses having widths larger than about L. This sets the lower band edge, for which one has full sensitivity, to be $\approx c/L$. Frequency stability of the master oscillator driving the Doppler system, and finite signal-to-noise ratio on the radio links, set the high frequency band limit to be ≈ 0.1 Hz.

The principal noise sources affecting Doppler tracking experiments are [2, 3]: ground electronics noise (such as frequency standard and frequency distribution noise) antenna mechanical noise, thermal noise due to finite signal-to-noise ratio on the radio links, spacecraft noise (such as electronics and unmodeled motion of the spacecraft) and phase scintillations introduced on the radio beams as they pass through irregularities in the troposphere, ionosphere, and the solar wind.

Electronic and thermal noises can be made very small, and propagation noises can be mitigated through use of higher or multiple radio frequencies to suppress or remove entirely charged particle scintillation, and by employing water vapor radiometers to measure and calibrate tropospheric scintillation. Residual uncalibrated troposphere and antenna mechanical noise are expected to be leading noise sources in the future gravitational wave experiments performed by Cassini [4], a scientific mission to Saturn sponsored by the National Aeronautics and Space Administration (NASA), the European Space Agency (ESA), and the Italian Space Agency (ASI). A significant effort from these agencies has been invested in the development and implementation of Ka-Band (≈ 32 GHz) capabilities, on board the spacecraft and on a ground radio telescope of the NASA's Deep Space Network, for making radio science experiments. Searches for gravitational radiation in the millihertz band by Doppler tracking are part of the radio science experiments that will take advantage of the Cassini Ka-Band radio link.

2. Doppler Transfer Functions

In the Doppler tracking technique a distant interplanetary spacecraft is monitored from Earth through a microwave tracking link, and the Earth and the spacecraft act as free falling test particles. A radio signal of nominal frequency ν_0 is transmitted to the spacecraft, and coherently transponded back to Earth. There it is received and its frequency is compared to the frequency generated by the highly stable clock that provides the frequency to the transmitted signal. Relative frequency changes $\Delta\nu/\nu_0$ as functions of time are then measured. A gravitational wave propagating through the solar system causes small perturbations in $\Delta\nu/\nu_0$ which are replicated three times in the Doppler data. This characteristic signature of the Doppler response, referred to as the *three-pulse* response, was first derived in its general form in 1975, by Estabrook and Wahlquist [1]. Its analytic expression can be written as follows

$$y_{GW}(t) = -\frac{(1-\mu)}{2}\, h(t) \ - \ \mu\, h\left(t - \frac{(1+\mu)}{2} T_2\right) + \frac{(1+\mu)}{2}\, h(t - T_2) \ , \quad (1)$$

where $h(t)$ is equal to

$$h(t) = h_{+}(t)\cos(2\phi) + h_{\times}(t)\sin(2\phi) \ . \quad (2)$$

Here $h_{+}(t)$, $h_{\times}(t)$ are the wave's two amplitudes defined with respect to two orthonormal axis (X, Y) chosen in the plane of the wave, (θ, ϕ) are the polar angles describing the location of the spacecraft with respect to the right-handed triad (X, Y, Z) (with Z orthogonal to the $X - Y$ plane), and μ is equal to $\cos\theta$.

Significant effort has been devoted at the Jet Propulsion Laboratory (JPL) to the understanding of the noise sources affecting spacecraft Doppler tracking, and to estimate their strengths and spectral properties [2, 3, 4, 5]. In what follows we will summarize the main noise sources affecting Doppler tracking, and specifically the Cassini experiments, by providing their one-sided power spectral densities, $S_{y_n}(f)$, with $y_n(t)$ being the relative frequency fluctuations associated with a noise source $n(t)$.

2.1. Instrumental Noise Sources

In the high frequency region of the band accessible to Doppler tracking, thermal noise dominates over all other noise sources at about 10^{-1} Hz. This noise is white in phase, being determined essentially by the finite effective temperature of the receiver and the finite intensity of the signal. In frequency the power spectral density therefore grows with the square of the Fourier frequency, $S_{y_{th}}(f) \propto f^2$, making this noise source the dominant one in the "blue" region. From the knowledge of the onboard and ground system noise temperatures, and estimate of the strength of the microwave signals received at the spacecraft and at the ground station, the one-sided power spectral density of the thermal noise, $S_{y_{th}}(f)$, expected to affect the Cassini experiments is equal to

$$S_{y_{th}}(f) = 1.9 \times 10^{-25}\ f^2\ \text{Hz}^{-1} \quad (3)$$

Since this noise appears at the moment of detection t, its transfer function into the Doppler data is the identity.

Among all other instrumental noise sources (transmitter and receiver, stability of the spacecraft transponder, spacecraft buffeting, micro seismic disturbances, instabilities introduced by signal distribution within the ground station, clock noise, electronics) clock noise has been shown to be the most important instrumental source of frequency fluctuations [4]. If we define $C(t)$ to be the random process associated with the relative frequency fluctuations introduced by the clock into the Doppler observable, it is easy to see that $C(t)$ shows up at two different times, namely under the linear combination $C(t-T_2)-C(t)$. This time signature can be understood by observing that the frequency of the signal received at time t contains clock fluctuations transmitted T_2 seconds earlier. By subtracting from the received frequency the frequency of the radio signal transmitted at time t, we also subtract clock frequency fluctuations with the net result shown above. If we denote with $S_C(f)$ the power spectral density of the noise $C(t)$, we deduce that the spectral density of the clock noise in the Doppler data has the following dependence on the Fourier frequency f:

$$S_{y_C}(f) = 4 \sin^2(\pi f T_2) \; S_C(f) \ , \tag{4}$$

with $S_C(f)$ given by the following expression [5]

$$\begin{aligned} S_C(f) &= 6.2 \; [10^{-26} f + 10^{-31} f^{-1} + 10^{-28}] \; Hz^{-1} \ , \ 10^{-5} \leq f \leq 5.7 \; 10^{-3} \text{ Hz} \\ &= \; 6.5 \; 10^{-30} \; f^{-1} \; Hz^{-1} \ , \ 5.7 \; 10^{-3} \leq f \leq 1 \text{ Hz} \ . \end{aligned} \tag{5}$$

2.2. Propagation Noise Sources

The radio link to the spacecraft crosses regions of space in which the index of refraction $n(t, \vec{r})$ is different from one, and changes in space and time. The propagation noise is due to fluctuations in the index of refraction of the troposphere, ionosphere, and the interplanetary solar plasma.

As the radio signal to the spacecraft crosses the Earth's troposphere, it suffers a path delay ΔL. The time variations of ΔL induce frequency shifts $\Delta\nu$, of the main carrier frequency ν_0:

$$\Delta L = \int_{raypath} [n(t, \vec{r}) - 1] \; ds \ \ ; \ \ \Delta\nu = \frac{\nu_0}{c} \; \frac{d}{dt} \Delta L(t), \tag{6}$$

where the main contribution to the integral is limited to a region around the Earth. It is important to say that at microwave frequencies the index of refraction of tropospheric irregularities does not depend on the carrier frequency ν_0.

Let us define $T(t)$ to be the random process associated with the frequency noise due to the troposphere. From the definition of the Doppler observable we have that $T(t)$ enters into the Doppler data at two different times through the linear combination $T(t - T_2) + T(t)$. This is because the frequency of the received signal is affected at the moment of reception as well as T_2 seconds earlier. Since the frequency of the signal generated at time t does not contain yet any of these fluctuations, it follows that $T(t)$

is positive-correlated at the round trip light time T_2. These considerations imply the following power spectral density of the troposphere noise in the Doppler data

$$S_{y_T}(f) = 4 \cos^2(\pi f T_2) \ S_T(f) \ , \tag{7}$$

where $S_T(f)$ is the one-sided power spectral density of the frequency fluctuations due to the troposphere. It should be noted that the frequency fluctuations due to the mechanical vibrations of the ground antenna, as well as those induced by the ionosphere, enter into the Doppler observable with the same transfer function of the atmosphere, since both are noise sources local to the Earth. In order to minimize the effects of the atmosphere, Cassini will take advantage of a specifically built water-vapor radiometer for calibrating about eighty percent of the atmosphere-induced fluctuations.

Ionosphere and interplanetary plasma have been the dominant noise sources in spacecraft Doppler tracking experiments analyzed so far, in which S-Band ($\approx$ 2 GHz) and/or X-Band ($\approx$ 8 GHz) microwave links were used. Since the plasma index of refraction at microwave frequencies scales as ν_0^{-2}, these noise sources can be suppressed by increasing the microwave frequency. Furthermore, by collecting data at solar opposition, that is to say when the Sun-Earth-Probe angle is larger than about 160^0, the plasma wind remains in the radio beam much longer, minimizing in this way the temporal variations in the index of refraction.

From plasma scintillation data [6] it has been shown that the spectral density of this noise has a power law dependence, $S_{y_P}(f) \propto f^{-2/3}$. At Ka-Band, the one-sided power spectral density of the noise due this dispersive noise is given by the following expression

$$S_{y_P}(f) = 3.7 \times 10^{-31} \ f^{-2/3} \quad \mathrm{Hz}^{-1} \ . \tag{8}$$

3. Anticipated Cassini Performance

From the considerations made in the previous section, to a good approximation we can write the Doppler response $y(t)$ in the following form

$$\begin{aligned} y(t) = y_{GW} + C(t - T_2) \ - \ C(t) \ + \ Th(t) \\ + T(t - T_2) \ + \ T(t) \ + \ P(t) \ . \end{aligned} \tag{9}$$

If we denote with $\widetilde{y}(f)$ the Fourier transform of $y(t)$ calculated over the time of observation 2τ, this is defined as follows

$$\widetilde{y}(f) = \int_{-\tau}^{\tau} y(t) \ e^{2\pi i f t} \ dt \ , \tag{10}$$

and at the Fourier frequencies

$$f_k = \frac{(2k-1)}{2T_2} \qquad ; \qquad k = 1, 2, 3, ... \tag{11}$$

Eq. (9) can be rewritten in the Fourier domain in the following approximate form

$$\begin{aligned} \widetilde{y}(f_k) \approx \left[-1 + i \ (-1)^k \ \mu \ e^{\frac{\pi}{2} \ i \ (2k-1) \ \mu}\right] \ \widetilde{h}(f_k) \ - \ 2 \ \widetilde{C}(f_k) \ + \ \widetilde{Th}(f_k) \\ + \ \widetilde{T}(f_k) \left(\frac{\pi i T_2}{4\tau}\right) \ + \ \widetilde{P}(f_k) \ . \end{aligned} \tag{12}$$

In Eq. (12) we have assumed that the time dependence of the two-way light time is such that during the time scale 2τ over which the Fourier transform is performed the frequencies f_k change by an amount smaller than the frequency resolution $\Delta f = 1/2\tau$. In the case of the Cassini trajectory it has been shown by Tinto and Armstrong [4] that the variation of the Earth-Cassini distance during the forty days of tracking implies a change in the frequencies f_k smaller than the frequency resolution Δf.

We note from Eq. (12) that the Fourier components of the random process T at the frequencies f_k are reduced in magnitude by the factor $(\pi T_2)/(4\tau)$. With a two-way light time of about fifty five hundred seconds, and an observation time of forty days, the reduction factor for the frequency fluctuations due to the troposphere, ionosphere, and the mechanical fluctuations of the ground antenna, is equal to 2.5×10^{-3} during the first Cassini opposition.

From Eq. (9) we can derive the expression for the expected one-sided power spectral density $S_y(f)$ of the noise into the Doppler response $y(t)$. Under the assumptions that the random processes associated with each noise source are uncorrelated with each other, and their one-sided power spectral densities are as given in the previous section, in Figure 1 we plot the estimated one-sided power spectral density of the noise, $S_y(f)$, for the Cassini experiments. The continuous-line curve corresponds to the configuration in which eighty percent of noise due to the troposphere has been calibrated out with water vapor radiometry, while the dotted-line assumes no calibration of the troposphere. Since the power of the frequency fluctuations due to the troposphere are reduced, at the frequencies f_k, to a level smaller than the remaining noise sources, we see why the two curves plotted in Figure 1 coincide at the frequencies f_k.

Figure 1 also provides the main element for estimating the signal-to-noise ratios Cassini might be able to achieve with various waveforms. A complete analysis of the Cassini sensitivities to bursts, continuous and stochastic signals has been performed by Tinto and Armstrong [4], and we refer the interested reader to them for a complete analysis. Here we briefly summarize the signal-to-noise ratios expected to be achieved by Cassini for the three classes of sources.

Gravitational wave bursts in the millihertz frequency band could be emitted during different astrophysical scenarios. A collapse of a star cluster to form a supermassive black hole, for instance, might generate a waveform whose dominant spectral components fall in the Cassini frequency band. Another astrophysical scenario implying the emission of a gravitational wave burst is the fall of small black holes into a super massive black hole, as it might happen at the end of the merger of two galaxies hosting at their centers a black hole. Although the temporal dependence of the gravitational wave burst radiated during the merger is unknown, the radiation emitted by the newly formed hole during the settling process is well known. For this class of sources Cassini is expected to achieve a relative frequency (strain) sensitivity of about 3×10^{-15}.

In the case of the stochastic background with bandwidth equal to center frequency, the sensitivity of the Doppler data at the frequencies f_k is given by the expected root-mean-squared (r.m.s.) noise level $\sigma(f_k)$ of the frequency fluctuations in the bins of width

f_k. This is given by the following expression

$$\sigma(f_k) = [S_y(f_k)\ f_k]^{1/2}\ \ ,\ k = 1, 2, 3, ...\ , \tag{13}$$

where $S_y(f_k)$ is the one-sided power spectral density of the noise sources in the Doppler response $y(t)$ at the frequency f_k, as given in Figure 1. In this case best sensitivity is achieved at the lowest "xylophone" frequency, f_1. For the first Cassini opposition, $f_1 = 9 \times 10^{-5}$ Hz. We get an energy density per unit logarithmic frequency and per unit critical energy density Ω equal to $\approx 10^{-2}$, after taking into account the effect of the r.m.s. antenna pattern. Subsequent Cassini oppositions have lower first frequency f_1, giving $\Omega \approx 4 \times 10^{-3}$ [4]. Cassini will give the best observational upper limit on a gravitational wave background in the millihertz band.

Finally, sinusoidal gravitational waves in the millihertz frequency band that happen to have their frequencies near one of the frequencies f_k might be observable by Cassini. At 9.0 $\times 10^{-5}$ Hz, and with an observation time of 40 days, the expected r.m.s. noise level at this frequency is equal to 5×10^{-17}.

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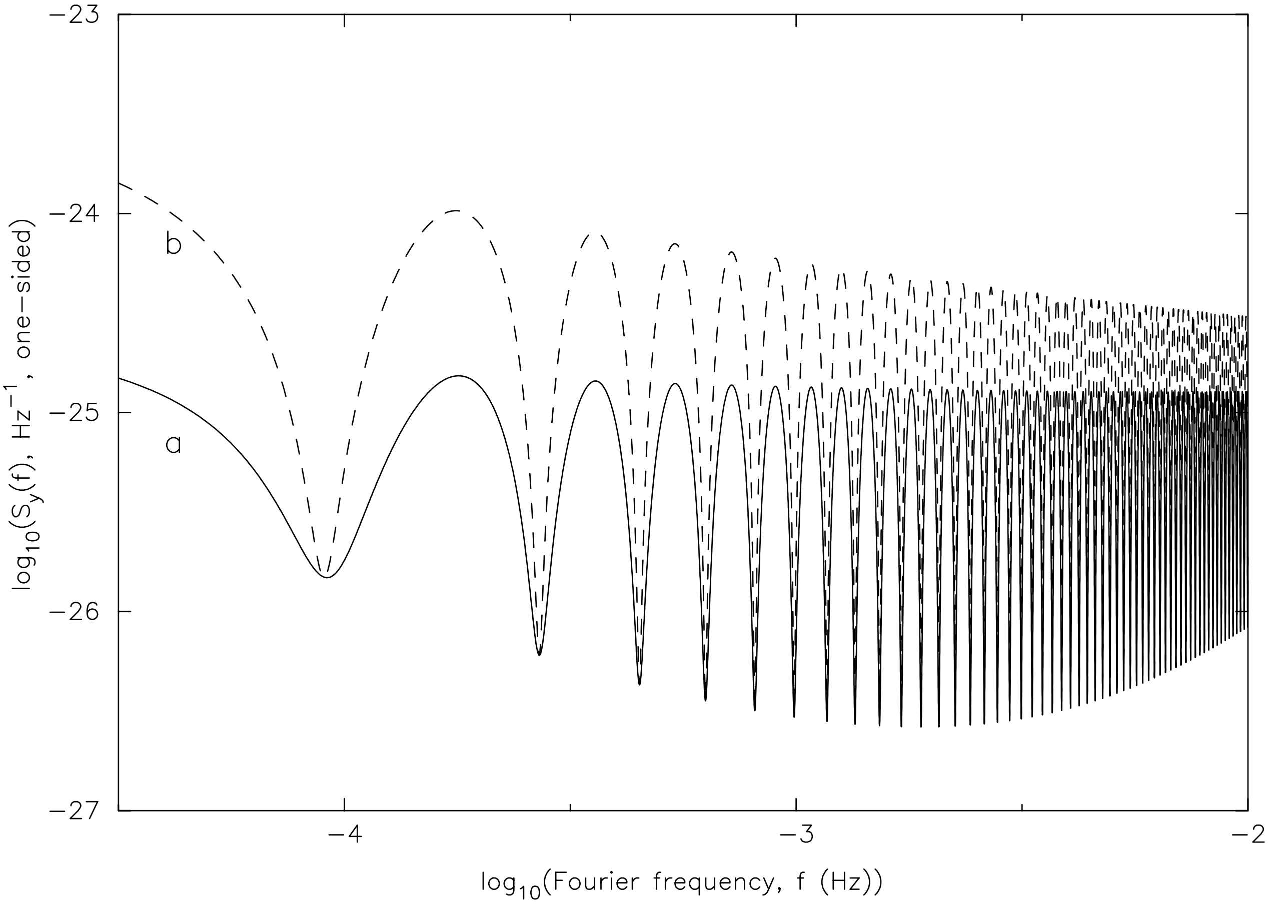

Figure 1. The estimated one-sided power spectral density of the noise that will affect the Cassini Doppler data. Curve "a" represents the configuration in which 80 percent of the noise due to the troposphere has been calibrated and removed by means of water vapor radiometry; curve "b" corresponds to the configuration without calibration of the troposphere.