Response of spherical gravitational wave antenna modes to high energy cosmic ray particles

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Abstract. High energy cosmic ray particles are expected to be a significant source of noise in resonant mass gravitational wave detectors close to the quantum limit. The spherical, fourth generation antennas are been designed to attain such limit. In this work we will show how the energy of a cosmic ray particle interacting with such an antenna is distributed over its eigenmodes. We will then make some comments on relevant consequences of such distribution to gravitational wave detection.

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1. Introduction

According to General Relativity (GR), gravitational waves are distortions of the space-time metric that propagate with the speed of light. The existence of such waves was predicted mathematically in 1916 by Albert Einstein, when he found them as radiant solution in vacuum for the equations of his theory of GR using a weak field approximation.

There are several resonant-mass gravitational wave detectors currently operating [1]. They have a cylindrical shape, which was chosen due to practical advantages. The next generation of resonant-mass detectors is expected to use antennas with spherical or buckyball shape [2] .This kind of detector will have larger mass than a cy1inder antenna tuned to the same frequency, which implies a greater cross-section. Also, the sphere is sensitive to waves arriving from all directions (omnidirectional) and, in case of monochromatic radiation, a single detector could determine the direction of the source as well as the wave's polarization [3,4] .

Several countries are now involved in the studies necessary for the construction of buckyball or spherical detectors, including Brazil. The Brazilian group is working on different programs aimed at the construction of, initially, a small detector, with 65 cm in diameter, using a copper-aluminum alloy [5] with 94% of Cu and 6% of Al.

In the effort to construct more sensitive bar and buckyball detectors there is a source of noise that is expected to become increasingly important: the interaction of cosmic rays with the antenna. In this paper we present some results on the investigation of the interaction of high energy particles with a spherical gravitational wave antenna.

2. Energy distribution

High energy cosmic ray particles hitting the resonant-mass antenna of a gravitational wave detector are expected to transfer energy to the antenna. Such energy eventually becomes a source of heat inside the material, and as this heat diffuses through the material it excites the vibrational normal modes of the antenna. These vibrations are measurable quantities for the detector and they can be confused with gravitational wave signals. It is therefore important to determine how the cosmic ray particle excites the normal modes of the antenna in order to separate its noise from the relevant gravitational wave signal. We have worked on this problem and determined such excitation pattern for the case of a spherical antenna. The details of the calculations will be published elsewhere[6].

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The expression we obtained for the energies deposited by the cosmic ray particle in each of the spherical antenna's modes is

$$E_{nlm} = \frac{1}{2M}\frac{\gamma^2}{\omega_{nl}^2}\left|\sum_{\nu=0}^{\infty} W_{nl\nu}\Theta_{\nu lm}\frac{1}{1-i\dfrac{\kappa\lambda_{\nu l}^2}{\rho c_\nu \omega_{nl}}}\right|^2 .$$

The indices n, l, and m are given by $n \in N^*$, $l \in N$, $m = [-l,+l]$, and they identify the different modes. The mass of the antenna is given by M, γ is the adimensional Gruneisen constant for the material, c_ν is its specific heat, κ is its thermal conductivity. The mode frequency is denoted by ω_{nl}, while λ_{nl} are constants characteristic of the respective modes.

The influence of the geometry on the energy absorption through each mode is contained in the respective constants $W_{nl\nu}$. The dependence of this function on the kind of particle, on the other hand, is given by the constants $\Theta_{\nu lm}$.

3. Cases of interest

We investigated two cases of interest to illustrate the use of the general analytical expression described above. Specifically, we obtained the distribution of energy through the modes in the cases when a cosmic-ray particle: 1) deposits energy in a point within the antenna and 2) crosses the antenna in a straight line while interacting with it.

3.1. Case 1: Interaction in a point

We located our reference frame (lab frame) in the geometrical center of the antenna with the *z* axis pointing upwards. Then in this simple case the particle interacts with the antenna only at the point $(\xi_{01}, \xi_{02}, \xi_{03})$ in the instant $\tau = 0$.

Figure 1 shows the example of a plot relative to the distribution of energy (in K) among different modes of a SCHENBERG-like antenna due to the deposition of energy by a high energy muon at a point located 10 cm from the center of the sphere. The SCHENBERG antenna is planned to be constructed by the Brazilian group under the GRAVITON project, with 65 cm in diameter antenna and made with a copper-aluminum alloy containing 94% of Cu and 6% of Al [7]. For illustration purposes only we assumed a hypothetical total energy loss of 3 GeV in a 60 cm-diameter antenna. The plot present energies in Kelvin for various modes *nl* with index m summed up from *-l* to *l*. In Table 1 explicit values for particular modes of interest are shown, also with the index m summed up from *-l* to *l*.

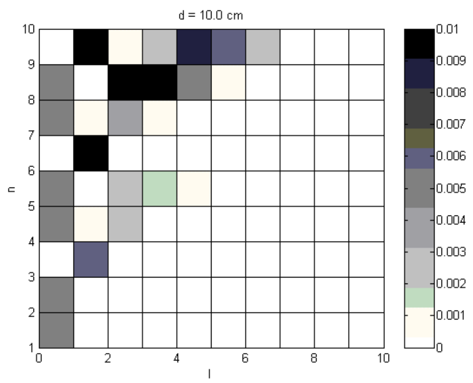

Figure 1. Energy distribution through different modes (μK) in the case of a cosmic ray particle that deposits all its energy in a point p located 10 cm from the center of a 60 cm-diameter sphere.

d (cm)	First monopole mode	First dipole mode	First quadrupole mode	Second quadrupole mode	Total energy deposited in the modes of Fig. 1
0.0	7×10^{-3}	0.0	0.0	0.0	2.9
10.0	5.2×10^{-3}	1.7×10^{-4}	2.8×10^{-6}	5.6×10^{-5}	0.2
20.0	1.8×10^{-3}	5.4×10^{-4}	4.5×10^{-6}	6.2×10^{-4}	0.4
29.6	1.2×10^{-3}	8.6×10^{-4}	1.7×10^{-4}	1.8×10^{-3}	0.2

Table 1. Energy deposition, in μK, for the case of energy release in a point distant *d* cm from the center of the sphere.

3.2. Case 2: Interaction along a straight line

In this case the particle's trajectory can be described by the Euler angles θ_0 and ϕ_0 plus the coordinates of its nearest point to the lab frame's origin, $p = (\xi_{01}, \xi_{02}, \xi_{03})$. As the cosmic ray particles travel essentially at the speed of light, delivering energy along a cylinder of negligible transverse size relative to the antenna's dimension, a relatively simple expression for the heat source can be used assuming, as usual, that: 1) the particle releases energy only along its trajectory, and 2) its speed is so high that for experimental purposes the energy release occurs virtually simultaneously along all the trajectory.

Figure 2 shows the plot for the energy released by a high energy muon through different modes of the SCHENBERG-like antenna, while crossing it in a straight line, the closest distance to the sphere's center being 10 cm, which corresponds to a total path length L = 56.6 cm. The explicit values for modes of special interest in gravitational wave detection are shown in Table 2.

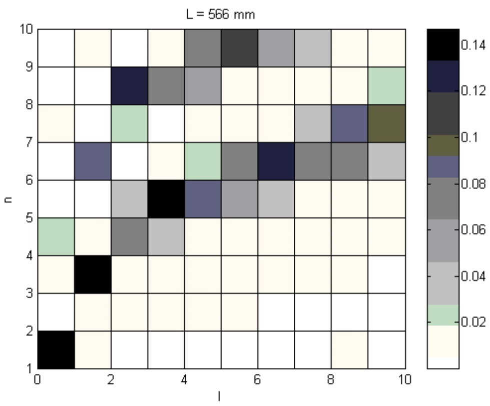

Figure2. Energy distribution through different modes (μK) in the case of a cosmic ray particle that deposits its energy along a straight line with closest distance to the center equal to 10 cm (L = 56.6 cm), assuming a 60 cm-diameter sphere.

L (cm)	First monopole mode	First dipole mode	First quadrupole mode	Second quadrupole mode	Total energy deposited in the modes of Fig. 2
60.0	2.1×10^{-1}	0.0	1.6×10^{-3}	2.2×10^{-2}	3.8
56.6	1.4×10^{-1}	9.1×10^{-3}	8.7×10^{-4}	1.1×10^{-2}	2.9
20.0	1.4×10^{-3}	6.3×10^{-3}	1.1×10^{-3}	1.2×10^{-2}	0.6
10.0	2.3×10^{-4}	1.7×10^{-3}	3.4×10^{-4}	3.6×10^{-3}	0.2

Table 2. Energy deposition, in μK, for the case of energy release along a straight line of length L cm.

4. Comments on the results

The main goal of this work was to report on the determination of a general expression for the energy distribution generated by a high energy particle in the various modes of oscillation of a spherical gravitational wave antenna. Such original expression was presented and applied to two particular cases of cosmic ray energy deposition patterns.

In gravitational wave detection the modes of greatest interest are the first monopole, the first dipole and the first and second quadrupole. The plots presented in the figures include these and many other modes so that we can have an idea about the pattern of excitation of the modes of a spherical antenna by a high energy particle.

In what concerns the detection of gravitational waves, we would like to point out that we noticed [6] that the first monopole mode is usually significantly excited unless the ionizing particle crosses a very short path close to the spherical antenna surface. This indicates that cosmic ray particles would probably generate noise if such mode is monitored in order to detect gravitational waves predicted by theories that admit scalar waves.

As for the first quadrupole mode, it is usually much less excited than the other modes of interest, specially the dipole mode. This result suggests that the first dipole mode could be used as a veto for cosmic rays in the detection of gravitational radiation as predicted by general relativity.

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