Drag-Free Performance in a LISA Mission with Spherical Proof Masses

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Abstract

The performance of a drag-free satellite with a spherical proof mass for the LISA space mission to detect gravity waves is presented. These calculations show that the RSS of all disturbances is less than 3.4×10^{-16} m/sec^2/Hz$^{1/2}$. This is about a factor of 10 less than the calculated value for cubical single-axis proof masses and could potentially decrease the low-frequency response correspondingly. The performance calculations presented here have been confirmed by the flight of the DISCOS drag-free satellite.

Introduction

The LISA mission to detect gravity waves from space requires a drag-free satellite [1] to reduce the disturbances on the proof masses to meet the low-frequency specification of 10^{-4} Hz. One possible design consists of free-floating spheres, completely unsupported, with a two cm gap between the proof mass and the cavity walls.

Since no forces or torques are deliberately applied to the proof mass, this design is simpler than a constrained single-axis proof mass and can potentially have lower disturbances. Torques must only be applied to the proof mass at the beginning of the mission to spin it to about 1 Hz and to actively damp the spin axis to be parallel to the sphere's maximum axis of inertia. The techniques for doing this are discussed in [2], and this paper will concentrate on the performance of the system described in that reference.

Performance Calculations

The LISA acceleration performance with a spherical proof mass is summarized in Table 1. The disturbances are divided into three categories:

1) Disturbances external to the spacecraft. These come from the interplanetary magnetic field and radiation of which the only important source is occasional solar flares.
2) Internal disturbances coupled to the relative motion of the spacecraft due to field gradients at the proof mass. These come from gravity, electric, and magnetic gradients and have been bounded by the DISCOS flight results [3].
3) Internal disturbances which do not depend on the relative motion of the spacecraft. These include gas-molecule collisions from the residual vacuum, gravity-gradient variations from thermal fluctuations, photon shot noise from the detectors and discharge UV, and a radiometer-like effect from temperature fluctuations in the proof mass.

External Disturbances

Since the only significant radiation source is solar flares (see Table 2) which are rare and can be removed from the data, the only important external disturbance is from the interplanetary magnetic field.

This has been measured by the Ulysses spacecraft [4]. This and other Ulysses data show that the interplanetary magnetic field power spectrum is $1/f^2$ with a peak of 10^4 nT^2/Hz at 10^{-4} Hz, i.e. the root power spectrum is $1/f$ with a peak of 100 $nT/Hz^{1/2}$ at 10^{-4} Hz. Because the theory of eddy currents in spinning conducting spheres is well understood [5] and because of the Ulysses measurements, the external disturbances are well in hand.

Internal Disturbances Coupled to the Spacecraft Relative Motion and DISCOS Bounds

In addition to the calculations summarized in Table 1, the DISCOS flight has demonstrated that no field gradient at the proof mass exceeded about 10^{-7} / sec^2. When this is combined with the expected drag-free performance of LISA, 10^{-9} $meters/Hz^{1/2}$; the DISCOS flight has demonstrated 10^{-16} $m/sec^2/Hz^{1/2}$ for the internal disturbances which depend on spacecraft motion.

Internal Disturbances which Do Not Depend on Spacecraft Motion

The major internal disturbance which does not depend on spacecraft motion is the specific force spectrum caused by collisions from gas molecules due to the residual vacuum. This is about 0.12×10^{-15} $m/sec^2/Hz^{1/2}$. This disturbance goes as the square root of pressure so even a major error in attaining the required vacuum would not have serious consequences.

The rest of the internal disturbances are very small so that an error in their estimation is unlikely to be important. Thus the conclusion is that these disturbances are also well in hand.

Source of Noise	Formulas	Auxiliary Formula, Critical Values, Comments, Etc.	Specific Force Noise
	DFS control = 10^{-9} m/Hz$^{1/2}$, a = 2.5 cm, gap = 2 cm, m_{PM} = 1.47 kg	T = 300 K, p = 10^{-9} Torr =1.33 × 10^{-7} N/m^2, ω_{PM} = 6 rad/sec	Units10^{-15} m/sec^2/Hz$^{1/2}$
Internal Magnetic Field Gradient	$\mathbf{f} = (\mathbf{m}_m \cdot \nabla)\mathbf{B}_{sc} / m_{PM}$	m_m = 4.2 × 10^{-9} Am^2, eddy currents. B_{sc} = 100 nT/Hz$^{1/2}$	2.8E-01
		m_m =7.2 × 10^{-11} Am^2, sus	4.9E-03
Lorentz Force from Interplanetary Field	$\mathbf{f} = Sh\, q\mathbf{v}_o \times \mathbf{B}_{ip} / m_{PM}$	Sh = 0.1, q = 6 × 10^{-13} coul, B_{ip} = 100 nT/Hz$^{1/2}$	1.3E-01
Electric Charge	Smythe, 5.08	q = 6 × 10^{-13} coul	3.2E-05
Delta T Fluctuations	Proof Mass T variations	ΔT_{PM} = 10^{-4} K/Hz$^{1/2}$	5.9E-02
Residual Gas	$\text{PSD} = 3\dfrac{2d_g kT}{m_{PM}^2}$	$d_g = 3pa^2\sqrt{\dfrac{2\pi\, m_{av}}{kT}}$	1.26E-01
Photon Gas	$\text{PSD} = 3\dfrac{2d_g kT}{m_{PM}^2}$	$d_{ph} = 1.11h\dfrac{a^2}{\lambda_m^4}$	7.76E-04
Cosmic Rays	See text.		4.90E-04
Solar Flares	See text. Not in RSS.	Can be omitted from data.	4.90E-01
Thermal Gravity	DISCOS meas × $\alpha\, \Delta T_{SC}$	ΔT_{SC} = 0.01 K/Hz$^{1/2}$	2.0E-04
Thermal Gravity Gradient		α =4 × 10^{-7} / K	3.5E-02
Motion Gravity Gradient	dg/dx = 3 × 10^{-8} / sec^2 from DISCOS meas and calc		3.0E-02
Transcollimator Light Pressure	$\text{PSD} = \dfrac{2h^2 n}{\lambda^2 m_{PM}^2} = \dfrac{2hP}{c\lambda\, m_{PM}^2}$	n = 5 × 10^{10} ph/sec P = 10^{-8} watts, λ = 10^{-6} m	5.72E-07
UV Discharge Light Pressure	$\text{PSD} = \dfrac{2h^2 n}{\lambda_{UV}^2 m_{PM}^2} = \dfrac{2hP}{c\lambda_{UV} m_{PM}^2}$	n = 6 × 10^5 ph/sec P =10^{-10} w, λ_{UV}=2 × 10^{-7} m	2.52E-08
Out-gassing Pumping Currents		No noise beyond residual gas	
Large Particles		All large particles assumed trapped with T = 0.15 sec	
	RSS of all sources	Units 10^{-15} m/sec^2/Hz$^{1/2}$	3.4E-01

Table 1. LISA Acceleration Error Budget for Spherical Free-Floating Proof Mass

Residual Gas	T K	p Torr	n #/cm3	R watts	d_g N/(m/s)	Q_g $m^2/s^4/Hz$
	300	1.0E-09	3.2E+07	1.2E+00	1.4E-12	1.6E-32
Photon Gas	T K	lambda m m	n #/cm^3	R watts	d_{ph} N/(m/s)	Q_{ph} $m^2/s^4/Hz$
	300	9.68E-06	5.50E+08	3.6E+00	5.25E-17	6.02E-37
Large-Particles	Av Vel m/sec	Collisions #/yr	Radius microns	P kg m/s	d_c N/(m/s)	Q_c $m^2/s^4/Hz$
Micron-Size	3.0E-02	100	0.5	3.7E-17	7.8E-21	1.8E-37
Unshielded Radiation	E mev	F $\#/cm^2/s$	P kg m/s	Δ P kg m/s	d_r N/(m/s)	Q_r $m^2/s^4/Hz$
Solar Flare	1.0E+02	1.0E+06	2.3E-19	2.3E-19	3.3E-20	2.4E-31
Solar Flare	3.0E+03	1.0E+02	1.3E-18	1.3E-18	3.3E-24	7.3E-34
Galactic Cosmic Rays	1.0E+02	1.0E+00	2.3E-19	2.3E-19	3.3E-26	2.4E-37
Iron Cosmic Rays	1.0E+06	5.0E-03	5.3E-16	1.7E-18	5.1E-31	1.9E-40

Table 2. Power Spectral Densities of Residual Gas, Large Particles, and Radiation

Assumptions and Notation

The notation in Table 1 is

a = radius of proof mass = 2.5 cm.
m_{PM} = mass of the proof mass = 1.47 kg.
ω_{PM} = angular velocity of the proof mass in rad/sec = 6 rad/sec.
m_m = magnetic moment in Amp-meters.
q = electric charge on the proof mass in coulombs.
f = specific force in $m/sec^2/Hz^{1/2}$.
B = magnetic field in Tesla (N/Amp-m).
PSD = power spectral density in $m^2/sec^4/Hz$.

The proof mass is assumed to be constructed of Osmium which is chosen because of its low electrical conductivity, $\sigma = 1.7 \times 10^6$ mhos/meter, and its high density, 22480 kg/m^3.

The electric charge is determined by assuming a potential of 0.1 Volt with a gap of 2 cm. The theory of the attraction between a charged sphere and a spherical cavity is developed in [5]. The eddy current theory is developed in detail in SUDAAR 194 [1] which is also on the web linked through [1]. It is assumed that the Lorentz force can be shielded by about a factor of 0.1.

The angular velocity of the spinning proof mass has been chosen at 1 Hz to keep variations due to proof-mass surface irregularities out of the LISA band.

Conclusion

Calculations of the acceleration disturbances of a free-floating spherical proof mass in the LISA experiment give an RSS value of 3.4×10^{-16} m/sec^2/Hz$^{1/2}$ which is about a factor of ten less than the value calculated for a cubical proof mass in a single-axis drag-free satellite [6]. This opens the possibility of reducing the LISA low frequency band by the same factor. Because DISCOS has demonstrated that the internal disturbances which depend on spacecraft motion are bounded at 10^{-16} m/sec^2/Hz$^{1/2}$ and because the other two classes of disturbance are well in hand, a technology demonstration flight to prove the drag-free performance would not be needed.

References

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[4] Balogh, A. et al., http://helio.estec.esa.nl/uds03/uls_08_a/DOCUMENT/MAG/MAGINST.HTM, Figure 9.

[5] Smythe, William R., *Static and Dynamic Electricity*, Third Edition, 5.08, p 128 (Charge forces) and 10.05, p 374 (Eddy currents), Hemisphere Pub. Corp., 1989.

[6] LISA Pre-Phase A Report, ftp://ftp.ipp-garching.mpg.de/pub/grav/lisa/ppa2.09.pdf and the LISA Status Report, ftp://ftp.rzg.mpg.de/pub/grav/lisa/sts_1.04.pdf, also www.dragfreesatellite.com/lisa.html, linked in paragraph 3.