

Low Loss Niobium Flexure Suspension Systems

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Abstract

Niobium membrane flexure suspensions have been proved to be able to achieve high pendulum Q-factor, while niobium cantilever suspensions have been proved to give high internal Q factors in sapphire test masses. Here we present the proposed sapphire test mass suspension systems based on the use of niobium flexures. This suspension system has the advantage of being robust while maintaining the high Q of the sapphire test mass. We show that it is also advantageous to use Nb flexures in future cryogenic detectors.

1. Introduction

Thermal noise sources are limiting noise sources in interferometer gravitational wave detectors. To reduce the internal thermal noise of the test mass and the pendulum noise of the suspension, very low loss materials should be used for the test mass and suspension. Current gravitational wave research projects use fused silica for test masses^[1,2]. It was proposed^[3] that sapphire, the lowest acoustic loss material of all, be used as test mass material for advanced interferometer detectors. There is now extensive research on sapphire materials^[4,5,6]. Present interferometer detectors use fused silica fibres^[7] as pendulum suspension material. Such suspensions result in good pendulum Q-factors^[8]. However fused silica material is fragile and prone to moisture damage. Furthermore, fused silica is non-compatible for future cryogenic detector. An alternative suspension scheme—niobium flexure suspensions has been proposed^[9]. Niobium membrane suspensions offer several advantages. First, niobium is the material with the lowest loss of all metals at all temperatures. Secondly, it is non-brittle and easy to use. It has excellent yield to Young's modulus ratio, which is crucial in high Q-factor flexure design. And finally, niobium has excellent cryogenic thermal conductivity which can be used effectively in cooling and maintaining low temperature for sapphire test masses. The short membrane design of the niobium flexure also has the advantage of avoiding violin string modes associated with long wire suspensions^[10].

2. Pendulum Q factor and membrane flexure

The Q-factor of a pendulum can be written as^[11]

$$Q_p = \frac{k_g + k_s}{k_s} Q_0 \approx \frac{k_g}{k_s} Q_0, \quad (1)$$

where k_g is the gravitational spring constant and k_s is the spring constant, depending on the geometry of the pendulum and the flexure respectively, while Q_0 is the intrinsic

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material Q-factor of the flexure. For a membrane flexure, k_s is described in angular spring constant term and is written as^[12]

$$\kappa_s = \frac{EI\lambda}{\tan(l\lambda)} \quad (2)$$

where E is the Young's modulus of the material, $I=at^3/12$ is the inertial of area of the membrane cross section, a is the width of the membrane and t is the thickness of the membrane, $\lambda = \sqrt{Mg/EI}$, l is the length of the membrane. For simplicity, we can assume that l satisfies the condition $l\lambda > 3$, so that $\tanh(l\lambda) \rightarrow 1$. Then, the angular spring constant is simplified to

$$\kappa_s = \sqrt{MgEI} \quad (3)$$

The gravitational spring constant is proportional to Mg , while in the case of a compound pendulum $\kappa_s=MgL$ where L is the distance to the center of mass. The pendulum Q-factor with membrane flexure can then be written as

$$Q_p = \sqrt{\frac{Mg}{EI}} L Q_0 \quad (4)$$

Substituting I into the equation (4) and using $\delta 1$ as the tensile loading factor (in addition to the bending stress), $\delta 1 Y = mg/at$, where Y is the yield strength, the above equation becomes

$$Q_p = \frac{L}{t} \sqrt{12\delta 1 \frac{Y}{E}} Q_0 \quad (5)$$

It can be seen that it is advantageous to have a thin membrane (while the width of the membrane can be relatively large to satisfy the strength requirement) to achieve high Q-factor.

Previous work has demonstrated that a monolithic niobium membrane flexure pendulum can achieve Q factor 8×10^6 (limited by the vacuum pressure)^[13].

3. Flexure module

To solve the practical problem of attaching the niobium flexure to the sapphire test mass, a flexure module was proposed. This module consists of micro-cantilevers and membrane as shown in Figure 1(b). This module will enable the flexure to be mounted and replaced with relatively ease. There are small tips on the cantilever at the supporting contact points. This is to provide high pressure contacts between the flexure and the test mass. Previous work with micro-cantilever suspension^[14] showed that high contact pressure suspension is important in achieving high Q-factor of sapphire samples.

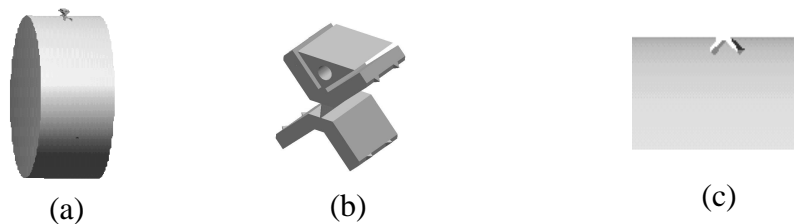


Figure 1. (a) Sapphire test mass suspended by niobium flexure module. (b) The flexure module consisting of a membrane incorporated with micro cantilevers. The overall dimension of the module is about 7mm x 7mm x 7mm. (c) Dovetail groove on sapphire test mass.

4. Effect of grooves on sapphire test masses

Use of the flexure module requires a small groove to be cut into the sapphire as shown in Figure 1. It is possible that such a cut might increase the thermal noise of the test mass. There are two main factors: (a) surface damage of the cut and (b) geometry effect of the cut --, that is the motion of the edge of the cut acting as extra resonant mass that couples to the test mass internal modes^[15]. We have demonstrated that the first factor can be minimised by annealing treatment of the sample after cutting. The later factor depends on the shape and position of the cut. Finite element modeling of a $\phi 150\text{mm} \times 60\text{mm}$ piece of sapphire with a dovetail cut shows that there are no significant extra mode introduced by the groove for modes below 100kHz. Modeling results also show that coupling of motion of the small mass around the cut to the motion of the middle of the mass is very small^[16].

We have tested the Q-factor of a small sapphire sample with a groove cut into it. The sample is a 30mm in diameter and 100mm long cylinder with two faces polished and fine ground sides. The measured Q-factor of this piece of sapphire without the cut is $\sim 1 \times 10^7$. Then a small straight groove $\sim 2\text{mm}$ deep and $\sim 0.5\text{mm}$ thick was cut in the middle of the cylinder. This caused the Q-factor to reduced to 8×10^5 . However, after low temperature annealing (1100 C), the Q-factor recovered to 1.3×10^7 .

Furthermore, a pair of dovetail grooves as shown in figure 1 were cut in the above sample. Unfortunately, significant imperfections and microcracks were introduced during this cut. The Q-factor degraded to 1.7×10^6 . After the low temperature annealing the Q-factor recovered to 7×10^6 . Similar results were obtained using double loop tungsten wire suspension as well as Nb flexure module suspension. This implies that the flexure module suspension does not create significant losses at this level. We note that for this test, the surface ratio between the grooves and the sample is large. It is useful to compare the large slotted sample with that required for the full scale test masses. The surface ratio between the small straight groove surface and the sample was 0.4%. It was 1.8% for the large dovetail grooves. For a 10 kg test mass the surface ratio between the test mass and the grooves would be $< 0.2\%$. Thus it is expected that with a good finishing and annealing the Q-degradation due to cutting in a full sized test mass should be very small.

5. Advantage of Niobium Flexure in Cryogenic Detector

In future cryogenic interferometer detectors, the short niobium membrane flexure module is advantageous. First, the cryogenic Q-factor of niobium is very high, about $10^7 \sim 10^8$ ^[17]. Secondly, niobium has excellent cryogenic thermal conductivity ($100\text{Wm}^{-1}\text{K}^{-1}$ @ 4K ^[18]). The short length of the membrane has very low thermal resistance. Since the contacting point of the niobium flexure cantilever is at yield, there is good contact between the flexure and the test mass. The thermal boundary resistance (Kapitza resistance) is given by

$$R_k = \beta / AT^3, \quad (6)$$

where A is the contacting area, β is the coefficient and T is the temperature. The value of the coefficient of β between solids is not well known, but is estimated to be $\beta \sim 10^{-3} \sim 10^{-4} \text{ m}^2\text{K}^4/\text{W}$ ^[19,20]. Assuming that the contact area A is the area under yield, then A is in the order of $5 \times 10^{-7} \text{ m}^2$ for a 10 kg mass. Using the worst case $\beta = 10^{-3}$, the

Kapitza resistance at 10K is about 2 K/W. This is small compared with the bulk thermal resistance of the Nb flexure (20K/W at 10K). We modeled the latter using the ANSYS finite element modeling package, for a flexure design optimized for a room temperature performance. The result of this analysis is given in Figure 3. The result is only negligibly modified by adding the Kapitza resistance, also shown in **Error! Reference source not found.** It can be seen that for a heat power input of 300mW, niobium flexure module is expected to maintain the test mass at 10K. For comparison the result of sapphire fibres suspension²¹ from TAMA group is also shown in figure 3.

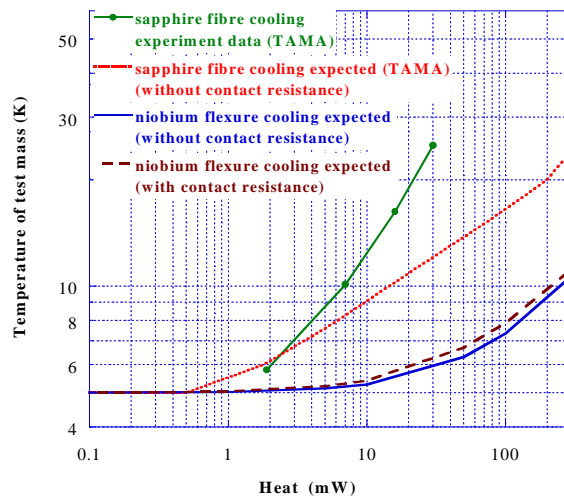


Figure 2 The expected cryogenic test mass temperature with different input heat power using niobium modular flexure. The top two lines are results from TAMA group with of sapphire fibre suspension.

The expected thermal noise of a 10kg sapphire test mass with Nb flexure suspension is shown in Figure 3. It can be seen that under cryogenic condition the pendulum thermal noise is close to the quantum limit.

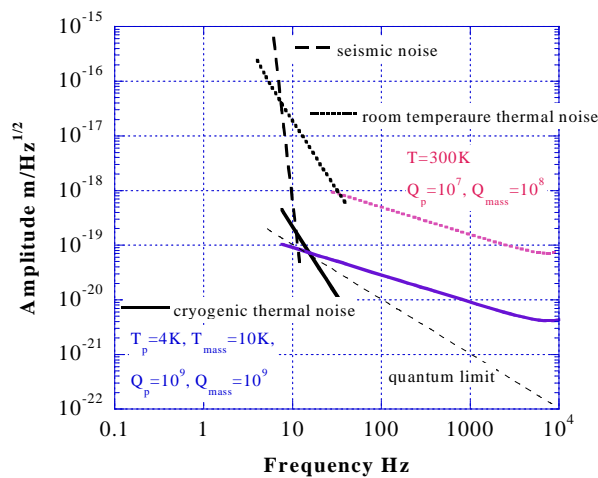


Figure 3. Comparison of thermal noise of room temperature and cryogenic interferometer gravitational wave detectors.

6. Conclusion

We have shown that sapphire test masses can be supported by niobium flexure modules without significant Q-degradation for Q values $>10^7$. Small grooves necessary to support the sapphire sample have a very small effect mode shapes of the test masses and do not degrade the Q-factor as long as the surface is annealed after cutting and as long as microcracks are avoided. At cryogenic temperatures niobium flexures offer a substantial advantage over other proposed suspension methods.

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