Cross-Correlation Studies with Seismic Noise.

H. Hoffmann, J. Winterflood, Y. Cheng, D.G. Blair

Department of Physics, University of Western Australia, Perth 6009, Australia.

Abstract

Ocean waves interacting in shallow water at the shore generate land waves propagating inland. To study these waves vertical, horizontal and tilt seismic noise were measured simultaneously at one location. Vibration isolators designed for gravitational wave research were used for detection. Cross-correlation was calculated between the above components. We found correlations between all of them. But, only the correlation between horizontal and vertical motion could possibly be addressed to land waves..

1 Introduction

Our motivation for studying cross-correlation on seismic noise was twofold. First, the isolation of seismic noise is a huge problem in the design and construction of gravitational wave detectors (see ref. [1] and references therein). Therefore, it is desirable to understand more about the nature of seismic noise. Second, there is, so far, only little experimental study done on Rayleigh waves (R-waves, sometimes called ground roll waves) as induced by ocean waves. This paper concentrates on the measurement of these surface waves.

R-waves can be generated by an oscillating pressure on the ground surface[2]. Two thirds of the generated energy is imparted into R-Waves and the other third in shear, and compression waves. We assume that the varying pressure of the incoming ocean waves as they enter shallow water, provides the driving pressure for their generation. Figure 1 illustrates the above process for ocean waves. This is born out by the remarkable correlation between microseismic signals in the 0.04 ~ 0.25 Hz band demonstrated by Liu[3] and Woo[4]. Figure 2 is a example of this correlation[4]. The resulting waves propagate in land as a surface wave. We will call this kind of wave "land waves". Little is known about their speed (we assume, it is around 1km/s).

In R-waves, the surface-particle motion (relative to a resting frame) describes a retrograde ellipse. The plane of the ellipse points in the direction of the wave propagation, and it is perpendicular to the surface of the ground. For a homogenous medium the ratio between vertical and horizontal component is unity at the surface and above unity below the surface[2]. We measured horizontal and vertical motion to identify ground roll.

For a Fourier component assuming that the size of the above ellipse is much smaller that the wave length, the vertical excitation (v) can be described by a standard wave equation,

$$v\,(x, t) = v_0 \sin(k\,x - \omega\,t) \tag{1}$$

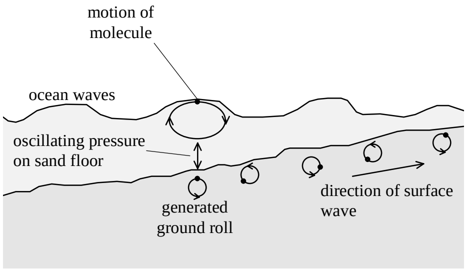

Fig. 1. The oscillating pressure on the sea bed induces a surface wave which propagates inland.

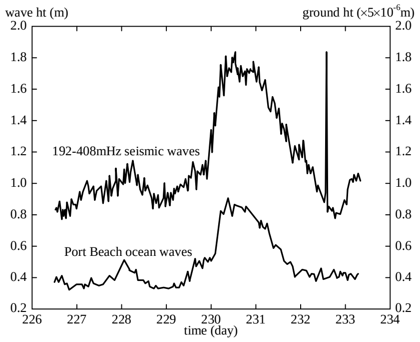

Fig. 2. Port Beach wave height (Aug 14-21, 9km South-West of UWA) and Micro-seisms at UWA

This results in the following tilt θ motion,

$$\theta (x, t) = v_0 k \cos(k x - \omega t) = \theta_0 \cos(k x - \omega t) \qquad (2)$$

There is a $\pi/2$ phase shift between tilt and vertical motion. If we were able to detect this phase shift the above relation would allow us to determine the speed c (equation 3) of the land waves by measuring vertical and tilt motion at the same position.

$$c = 2\pi f v_0/\theta_0 \qquad (3)$$

An alternative method of measuring the tilt would be to measure the wave travel time between two reasonably separated points. In This case one needs only to measure a single component of the wave. In this paper we will present methods for the correlation of v and θ at one point.

Methods

We measured the vertical, horizontal, and tilt component of seismic motion with three independent suspension systems. For the vertical motion, a LaCoste[5] pre-isolator stage was used, as designed for the pre-isolation system[6] for the Australian International Gravitational Observatory (AIGO). This pre-isolator was operated in a vacuum tank (10^{-1} torr) and its characteristics are shown in table 1. In principle a LaCoste isolator is equivalent to a simple spring mass system. The vertical position was read out with a shadow sensor (dynamic range 10mm) and it was servo controlled to the centre of its readout with a simple integral circuit with a very long time constant (RC > 1000sec). The output was fed back as a current in the suspending springs providing control of the vertical position by thermal weakening of the modulus of the springs - counteracting the springs intrinsic temperature sensitivity. The large time constant was obtained with the aid of a 1 Farad supercap.

The horizontal motion was measured with a Scott-Russel pendulum[7] which is equivalent to a very long simple pendulum (here 42 m). A tubular design was used similar to one leg of figure 5 of ref [7] with a simple weight on the top and using the taut wire vertical motion constraint at the bottom of the beam as per figure 2 of [7]. The whole movable structure was placed in an aluminium tube which functioned as the basic frame and protected the structure from air movements. The position was read out with a 2-D shadow sensor with a dynamic range of 1mm. Both north-south and east-west directions were recorded. Very long time-constant analog integrators were also used to stabilise the Scott-Russel pendulum since it has a tendency to drift. The servo actuation was applied by coils, one for each axis, with a small permanent magnet for each coil mounted on the suspended mass. A Scott-Russel pendulum does not purely measure horizontal vibration. Pendulum displacement occurs as a result of a combination of tilt and horizontal motion. But, as will be seen from the resulting spectra the tilt component was small enough to be neglected.

The tilt was measured using a double flexure inverse pendulum[8,9,10]. A massive disc (50 kg) is hanging from a rod. The disc is attached to the rod by a 2-dimensional flexure with centre of rotation slightly below its center of mass. This provides some inverse pendulum to counteract the spring-rate of the flexure. Another flexure connects the top of the rod to the frame. The displacement of the disc's rim due to tilt was read out with shadow sensors, one for each direction (north-south and east-west). No integrator feedback was required for equilibrium position control. Table 1 shows the characteristics of all three devices.

All three devices were located in the basement of the department of physics (on the Crawley Campus, Perth). Vertical, horizontal, and tilt were recorded simultaneously using LabView on a PC equipped with a Lab-PC+ (National Instruments) data acquisition board. Recording started on Friday the 29th of June 01 at 6.45 pm and lasted for 9.8 hr, sampling at 10Hz. Probably, no person entered the basement during that time.

	LaCoste (vert.)	Scott-Russel (horiz.)	Tilt sensor
Period (s)	15.5	13	17
Q-factor	4.5	4	10
susp. mass (kg)	370	2.53	50

Table 1. The table shows the characteristics of the three vibration isolation system used to measure the three components (of interest) of seismic noise.

Seismic noise spectra

From the obtained data, seismic noise spectra were calculated. The whole data array was split into files of length 4096 samples (corresponding to 409.6 sec). On each of the files a FFT was performed, and then the absolute value of all FFTs were averaged over all files. The resulting spectra were smoothed by averaging over a window of 5 adjoining points.

The transfer function T_f (detected vibration vs real vibration as a function of frequency f) for a simple pendulum or spring mass system is readily shown to be

$$T_f(f) = \left\| \frac{f^2 Q}{(f^2 - f_0^2)Q - \mathrm{i} f_0 f} \right\| \tag{4}$$

where Q is the quality factor and f_0 is the resonance frequency of the isolator. Both spectra of vertical and horizontal motion were corrected with the above transfer function using the values for Q and f_0 from table 1.

Cross-Correlation

On the same data cross-correlations between the different vibration measurements were calculated. Equation 5 defines the correlation function used.

$$\mathrm{Corr}(\tau) = \frac{\sum_n a(t_n) b(t_n + \tau)}{\sqrt{\sum a(t_n)^2 \sum b(t_n)^2}} \tag{5}$$

where $a(t_n)$ and $b(t_n)$ are the two data arrays, τ is the time shift between the two arrays. As above, the data was split into files of 4096 samples, and cross-correlations were averaged over these files. Before calculating cross-correlations the signals were frequency filtered, using a 0.04 to 0.2 Hz band. This band included the seismic spectra of interest. Outside this band the signals appeared to be very noisy and unreliable. The band was left reasonably broad to avoid forcing the signal into a harmony.

Results

The resulting seismic spectra is shown in Figure. 3A. The spectra suggests a connection between vertical and horizontal vibration. Between 0.1 and 0.2 Hz the vertical and horizontal displacement curves show similar characteristics. For two frequencies (0.095 and 0.145 Hz) both vertical and horizontal curves show maxima.

In Figure 3B the tilt noise spectra is shown. No transfer function correction was performed for the tilt. The measured transfer function gave values of around 1 (± 20%) for frequencies above 0.2 Hz, and 10 (± 10%) below 0.05 Hz. The measured transfer function did not match the calculated function (for detail see ref [10]).

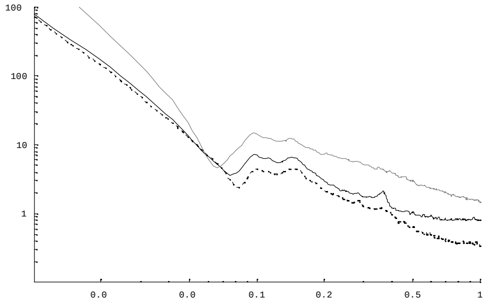

Fig. 3A Measured seismic displacement in μm/√Hz (solid:N-S, dashed:E-W, grey:vert.), frequency in Hz

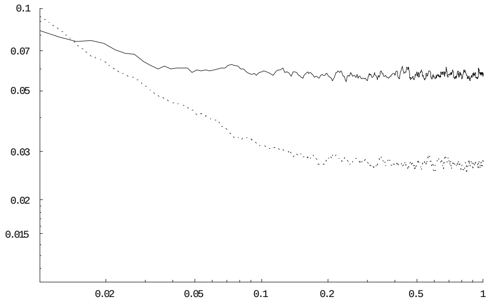

Fig. 3B Measured seismic tilt in μrad/√Hz (solid: N-S, dashed E-W), frequency in Hz

From the obtained spectra it could be deduced that the Scott-Russel unit was measuring mostly horizontal motion. At 0.1 Hz the measured tilt noise per √Hz was 6×10^{-8} rad (Figure. 3B, north-south) which results in a displacement on the Scott-Russel of 2.5μm (this was calculated by assuming that the isolator was equivalent to a simple pendulum of 42 m length). The value 2.5μm has to be compared with the measured 8μm displacement (fig. 3A, north-south). For higher frequencies this relation shifts towards a lower tilt contamination because the tilt transfer function drops as $1/f^{2}$ while the T_f for horizontal motion is approaching a constant value, 1 (see equation 4).

Figure 4 shows the correlation between the seismic vertical, horizontal and tilt motion. The resulting correlations showed a time delay between the horizontal and vertical motion of 1.0s (Figure. 4A and B). But, there was no time shift between vertical and tilt motion (Figure. 4C and D). There was no significant difference observed between the north-south and the east-west direction.

Discussion

The time delay (of 1s) between and the corresponding spectra of the vertical and horizontal vibration suggests the detection of ground roll, possibly, as from land waves. But, the phase

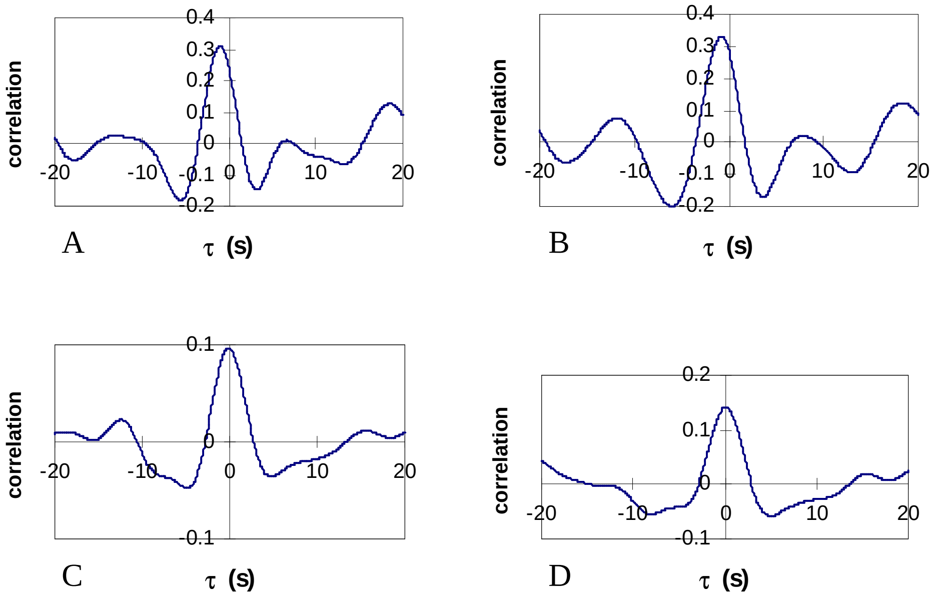

Fig. 4. Cross-correlation results of seismic noise between (A) vertical and north-south horizontal, (B) vertical and east-west horizontal, (C) vertical and north-south tilt, and (D) vertical and east-west tilt. Here τ is the time shift between captured signals. A time delay of 1.0s was observed for horizontal *vs* vertical. But, negligible phase shift was observed between vertical and tilt noise. Before calculating correlations the signal was frequency filtered using a 0.04 - 0.2Hz band.

shift represented by this time delay is smaller than the expected $\pi/2$ for ground roll (period of land waves: 6-8 s). A possible explanation might be that surface particles move (relative to a resting frame) along an elliptic curve rather than circular (with is also suggested by the difference in amplitude between vertical and horizontal - see Figure. 3A) and that this ellipse is tilted by some angle. This is illustrated in Figure. 5A. This motion can be regarded as a generalisation of the circular ground roll. The horizontal displacement is larger for the north-south direction that for the east-west direction (Figure. 3A). This would indicate a direction of wave propagation closer to north-south that to east-west (while the shore is west of Crawley). It is unclear if this is possible.

The negligible phase shift between vertical and tilt differs significantly from the expected $\pi/2$ phase shift for land waves (see equation 2). The measured tilt floor is higher than the expected tilt value due to a land wave. Assuming a wave length of 6 km, and a vertical displacement of 10 μm per $\sqrt{\text{Hz}}$ (Figure. 3A) the result is a tilt of 10^{-8} rad per $\sqrt{\text{Hz}}$. This value is significantly lower than the measured 3—6×10^{-8} rad per $\sqrt{\text{Hz}}$ (Figure. 3B). Thus, this suggests that some other source is responsible for the observed correlation. Figure 5B gives a possible explanation. The building containing the experiment could be shaking as a whole (e.g. as a result of wind). This movement has zero phase shift between tilt and vertical motion.

We could not determine the speed of the land waves suggested by equation 3. A more promising method seems to be the study of cross-correlation with horizontal excitations measured at distant locations. As an alternative one might perform an equivalent experiment outside the building.

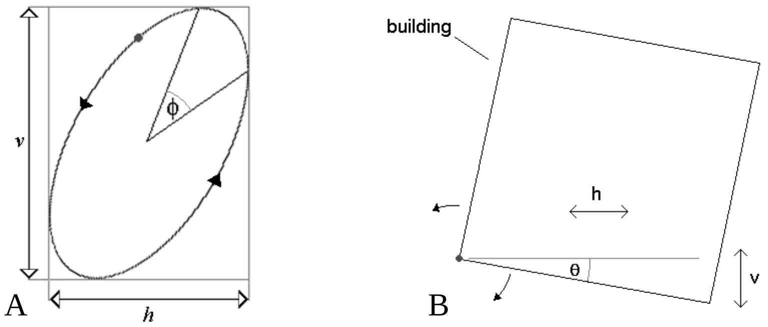

Fig. 5. In A, the motion of a ground particle relative to a resting frame is shown as expected for a land surface wave. The tilt of the ellipse is a generalisation, which allows the phase shift ϕ between vertical (v) and horizontal (h) motion to be less than a quarter of the wave period (as observed). In B, a possible explanation is given for the zero phase shift between vertical and tilt θ. The building in which the experiment was located vibrates as a whole, e.g., due to wind.

Acknowledgments

The first author (H. Hoffmann) wishes to thank all the members of the gravity wave research group at the UWA for their help and collaboration, and especially thanks D.G. Blair for his constructive ideas and for providing the basis of all the research done in the group. This work was supported by the Australian Research Council, and is part of the research program of the Australian Consortium for Interferometric Gravitational Astronomy.

References

[1] L. Ju, D.G. Blair, C. Zhao, *Rep. Prog. Phys*., **63,** 1317, (2000)

[2] F.E. Richart, J.R Hall, R.D. Woods, *Vibrations of Soils and Foundations* (1970), Prentice-Hall International Series, Chapter 3.

[3] J.F. Liu, D. G. Blair, and L. Ju, *Meas. Sci. Technol*. **9**, 1772 (1998)

[4] L.M. Woo, *Wave measurements using a micro-seismometer*. UWA Honours thesis (1998)

[5] L. LaCoste, *Geophysics*, **48**, 606 (1983)

[6] J. Winterflood, Z.B. Zhou, D.G. Blair, *Ultra-low Residual Motion Suspension System*, in *Proceedings of the 2nd TAMA International Workshop on Gravitational Wave Detection held at National Olympics Memorial Youth Center, Tokyo, Japan on October 19-22, 1999, Gravitational Wave Detection II*, (Universal Academy Press, Tokyo), p301-310, (2000).

[7] J. Winterflood, D.G. Blair, *Phys. Lett. A* **222,** 141,(1996)

[8] J. Winterflood, Z.B. Zhou, L. Ju, D.G. Blair, *Phys. Lett. A*, **277,** 143 (2000)

[9] Z.B. Zhou, J. Winterflood, L. Ju, D.G. Blair, *Phys. Lett. A*, **280**, 197 (2001).

[10] Y. Cheng, “Tilt sensor and servo control system for gravitational wave detection”, this proceeding