

Bounding the graviton mass with binary pulsar observations

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Abstract

By comparing the observed orbital decay of the binary pulsars PSR B1913+16 and PSR B1534+12 to that predicted by general relativity due to gravitational-wave emission, we are able to bound the mass of the graviton to be less than $7.6 \times 10^{-20} \text{ eV}/c^2$ at 90% confidence. This is the first such bound to be derived from dynamic gravitational fields. It is approximately two orders of magnitude weaker than the static-field bound from solar system observations, and will improve with further observations.

I. INTRODUCTION

General relativity assumes that the graviton has zero rest mass. For static gravitational fields a nonzero graviton mass m would cause the potential to tend to the Yukawa form $r^{-1}e^{-mr}$, effectively cutting off gravitational interactions at distances larger than the Compton wavelength m^{-1} of the graviton. Current experimental limits on the graviton mass are based on the apparent absence of such a cutoff in the solar system [1] and in galaxy and cluster dynamics [2,3].

The study of dynamic gravitational fields allows new and independent limits to be placed on the graviton mass. For example, Will [4] and Larson and Hiscock [5] have shown how future observations of gravitational waves may be used to bound the graviton mass via comparison to the dispersion formula

$$v(\omega) = \sqrt{1 - \frac{m^2}{\omega^2}}. \quad (1.1)$$

Here we propose a new technique for limiting m using available data on the orbital decay of binary pulsars. Consider the Hulse-Taylor binary pulsar, PSR B1913+16, for which the observed orbital decay attributed to gravitational-wave emission agrees with the predictions

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of general relativity to approximately 0.3%. A nonzero graviton mass would alter the energy emission rate and destroy this agreement; for gravitational waves at twice the orbital frequency of PSR B1913+16, requiring

$$\left(\frac{m}{\omega}\right)^2 < 0.003 \quad (1.2)$$

implies $m < \mathcal{O}(10^{-20}) \text{ eV}/c^2$. This is comparable to the limit $m < 4.4 \times 10^{-22} \text{ eV}/c^2$ obtained from solar system observations [1].

In this paper we develop this bound in more detail. In Section II we examine linearized general relativity with a massive graviton. In Section III we calculate the corrections to the energy emission rate of a compact, slowly moving source due to a nonzero graviton mass. Comparison to the observed orbital decay of PSR B1913+16 and PSR B1534+12 will provide us with an upper limit on the graviton mass in Section IV. We conclude in Section V with a few brief comments.

II. LINEARIZED GENERAL RELATIVITY WITH A MASSIVE GRAVITON

Gravitational waves on a flat background spacetime can be described as a perturbation of the Minkowski metric:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad |h_{\mu\nu}| \ll 1. \quad (2.1)$$

For convenience we work with the trace-reversed metric,

$$\bar{h}_{\mu\nu} \equiv h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h^\lambda{}_\lambda, \quad (2.2)$$

and raise and lower indices using $\eta^{\mu\nu}$, $\eta_{\mu\nu}$. Substituting (2.1) into the Einstein field equations, expanding in powers of $\bar{h}_{\mu\nu}$, and keeping only terms up to first order yields

$$\square \bar{h}_{\mu\nu} = -16\pi T_{\mu\nu}, \quad (2.3)$$

where the stress tensor $T_{\mu\nu}$ of the source matter is conserved, $T_{\mu\nu}{}^{;\nu} = 0$, and we work in the Lorentz gauge, defined by $\bar{h}^{\mu\nu}{}_{;\nu} = 0$.

A simple generalization of (2.3) to include a mass for the graviton is [8]

$$(\square - m^2)\bar{h}_{\mu\nu} = -16\pi T_{\mu\nu}. \quad (2.4)$$

The massive field $\bar{h}_{\mu\nu}$ has six degrees of freedom: five spin-2, and one scalar [8,9]. It is not gauge invariant, but the Lorentz condition $\bar{h}^{\mu\nu}{}_{;\nu} = 0$ still holds as a constraint.

An effective stress tensor for the gravitational waves can be obtained using Noether's theorem [10], and is identical in form to the usual $m = 0$ result [11]:

$$T_{\mu\nu}^{\text{GW}} = \frac{1}{32\pi} \langle \bar{h}_{\alpha\beta,\mu} \bar{h}^{\alpha\beta}{}_{,\nu} - \frac{1}{2} \bar{h}^\alpha{}_{\alpha,\mu} \bar{h}^\beta{}_{\beta,\nu} \rangle. \quad (2.5)$$

Here the brackets denote an averaging over at least one period of the gravitational wave.

III. ENERGY EMISSION RATE

We now calculate the energy emission rate of compact, slowly moving sources of gravitational waves described by (2.4)–(2.5). Our analysis follows that used for standard linearized general relativity [11], except that we work in the frequency domain for convenience [12].

For a periodic source with period P , we decompose the metric perturbation as

$$\bar{h}_{\mu\nu}(t, \vec{x}) = \sum_{n=-\infty}^{\infty} \tilde{h}_{\mu\nu}(\omega_n, \vec{x}) e^{-i\omega_n t}, \quad (3.1)$$

where

$$\omega_n = n \frac{2\pi}{P} \quad (3.2)$$

with n an integer. Then (2.4) becomes

$$(\nabla^2 + [\omega^2 - m^2]) \tilde{h}_{\mu\nu}(\omega|\vec{x}) = -16\pi \tilde{T}_{\mu\nu}(\omega|\vec{x}), \quad (3.3)$$

where ∇^2 is the 3-space Laplacian. The retarded Green function for this equation is

$$\tilde{G}_R(\omega|\vec{x}; \vec{x}') = \frac{e^{ik|\vec{x}-\vec{x}'|}}{4\pi|\vec{x}-\vec{x}'|}, \quad (3.4)$$

where $k \equiv \text{sign}(\omega)\sqrt{\omega^2 - m^2}$ for $|\omega| > m$. The retarded solution of (3.3) for fixed ω is then

$$\tilde{h}_{\mu\nu}(\omega|\vec{x}) = 16\pi \int d^3x' \tilde{G}_R(\omega|\vec{x}; \vec{x}') \tilde{T}_{\mu\nu}(\omega|\vec{x}'). \quad (3.5)$$

Using the slow-motion approximation ($\omega a \ll 1$, with a the characteristic size of the source), taking the observation point far from the source region ($r \equiv |\vec{x}| \gg a$), and employing the conservation of the stress tensor, one obtains

$$\begin{aligned} \tilde{h}_{00}(\omega|\vec{x}) &= \frac{4e^{ikr}}{r} \left[\tilde{M} + \frac{x^j}{r} (-ik) \tilde{D}_j + \frac{x^j x^k}{2r^2} (-ik)^2 \tilde{I}_{jk} \right], \\ \tilde{h}_{0j}(\omega|\vec{x}) &= \frac{4e^{ikr}}{r} \left[-(-i\omega) \tilde{D}_j - \frac{x^k}{2r} (-ik) (-i\omega) \tilde{I}_{jk} \right], \\ \tilde{h}_{jk}(\omega|\vec{x}) &= \frac{4e^{ikr}}{r} \left[\frac{1}{2} (-i\omega)^2 \tilde{I}_{jk} \right], \end{aligned} \quad (3.6)$$

where \tilde{M} , \tilde{D}_j , \tilde{I}_{jk} , are respectively the Fourier coefficients of the mass, dipole moment, and quadrupole moment of the source. Only the quadrupole terms contain nonzero-frequency components and contribute to the radiation.

Substituting (3.6) into the effective stress tensor (2.5) for the gravitational waves and integrating the outward flux over a sphere centered on the source gives the rate of energy emission. One finds

$$L \equiv -\frac{dE}{dt} = L_{\text{GR}} + \sum_{n=1}^{\infty} \frac{m^2 \omega_n^4}{3} \left[\tilde{I}_{jk}(\omega_n) \tilde{I}_{jk}^*(\omega_n) - \left| \text{tr} \tilde{I}(\omega_n) \right|^2 \right] + \mathcal{O}(m^4), \quad (3.7a)$$

where

$$L_{\text{GR}} \equiv \sum_{n=1}^{\infty} \omega_n^6 \left[\frac{2}{5} \tilde{I}_{jk}(\omega_n) \tilde{I}_{jk}^*(\omega_n) - \frac{2}{15} \left| \text{tr} \tilde{I}(\omega_n) \right|^2 \right] \quad (3.7b)$$

is the usual general-relativistic expression for the radiated power, $\text{tr} \tilde{I}$ is the trace of \tilde{I}_{jk} , and we sum over repeated indices. Equation (3.7a) gives the corrections to the radiated power due to a small nonzero graviton mass; comparison to the observed orbital decay in binary pulsars PSR B1913+16 and PSR B1534+12 will provide us with a bound on m .

IV. BINARY PULSARS

The formula (3.7) for the energy-loss rate of a gravitational-wave source when the graviton is massive is easily applied to the orbital decay of binary systems to put a limit on the graviton mass. Consider PSR B1913+16, for which the orbital decay rate is slightly in excess of the predictions of general relativity [6]. Denote by P_b the measured orbital period of the binary system, \dot{P}_b the measured orbital period derivative ascribed to gravitational radiation, and \dot{P}_{GR} the instantaneous period derivative expected from general relativity. For a slowly decaying Keplerian binary, the instantaneous period derivative is proportional to the energy-loss rate; hence,

$$\Delta \equiv \frac{\dot{P}_b - \dot{P}_{\text{GR}}}{\dot{P}_{\text{GR}}} = \frac{L - L_{\text{GR}}}{L_{\text{GR}}}, \quad (4.1)$$

where L is the gravitational-wave luminosity inferred from \dot{P}_b , and L_{GR} is the energy-loss rate expected from general relativity. This fractional discrepancy Δ has been measured for PSR B1913+16 and PSR B1534+12 (see [6,7] and Table I).

Now suppose that Δ is due at least in part to a nonvanishing graviton mass. Combining (3.7) and (4.1) implies

$$m^2 \leq \frac{24}{5} F(e) \left(\frac{2\pi\hbar}{c^2 P_b} \right)^2 \frac{\dot{P}_b - \dot{P}_{\text{GR}}}{\dot{P}_{\text{GR}}}, \quad (4.2)$$

where $F(e)$ is a function of the eccentricity,

$$F(e) \equiv \frac{\frac{1}{12} \sum_{n=1}^{\infty} n^6 \left[3\tilde{I}_{jk}(\omega_n) \tilde{I}_{jk}^*(\omega_n) - \left| \text{tr} \tilde{I}(\omega_n) \right|^2 \right]}{\sum_{n=1}^{\infty} n^4 \left[\tilde{I}_{jk}(\omega_n) \tilde{I}_{jk}^*(\omega_n) - \left| \text{tr} \tilde{I}(\omega_n) \right|^2 \right]} = \frac{1 + \frac{73}{24}e^2 + \frac{37}{96}e^4}{(1 - e^2)^3}, \quad (4.3)$$

as can be shown using the techniques of [13]; see Figure 1.

Equations (4.2), (4.3) show that a nonzero graviton mass increases the energy emission and decay rate of Keplerian binaries, as one would expect from adding extra degrees of freedom to the gravitational field. The strongest bounds arise from binaries with small eccentricity and large period, as these systems produce the bulk of their radiation at low frequencies [13], which are the most sensitive to a graviton mass, as in equation (1.1).

In using (4.2) to place an upper limit on the graviton mass, we should take into account the experimental uncertainties in the fractional discrepancy Δ , which are typically of the same order as Δ . We assume the measured discrepancy Δ to be normally distributed about its unknown actual value (given by the equality in (4.2) with unknown m^2), and with standard deviation as given in Table I. In our model we relate the discrepancy to the squared graviton mass, which must be non-negative. Referring to [14, Table X], which lists the 90% unified upper limit/confidence intervals for the non-negative mean of a univariate normal distribution based on a measured sample from the distribution, we calculate the 90% upper limit on the graviton mass, which is given in the final row of Table I. These two observations of m^2 may also be combined into a single upper bound on the graviton mass by averaging the individual m^2 bounds with weight according to their variances. Again referring to Table I and [14, Table X], the corresponding limit on the graviton mass from the combined observations of PSR B1913+16 and PSR B1534+12 is found to be

$$m_{90\%} < 7.6 \times 10^{-20} \text{ eV}/c^2. \quad (4.4)$$

V. DISCUSSION

Table I gives the relevant parameters and the corresponding graviton mass bounds for the two binary pulsars whose gravitational-wave induced orbital decay has been measured, PSR B1913+16 and PSR B1534+12 [6,7]. These bounds are about two orders of magnitude weaker than the Yukawa limit obtained from solar-system observations, $mc^2 < 4.4 \times 10^{-22}$ eV [1], and several orders of magnitude weaker than that provided by observations of galactic clusters, $mc^2 < 2 \times 10^{-29}$ eV [2,3], though these galactic cluster bounds may be less robust, owing to their reliance on assumptions about the dark matter content of the clusters. In contrast, the bound obtained here is very straightforward. Our chief assumption is the form of the effective mass term for the graviton, which, while not unique, is natural. Furthermore, any other mass term would be expected from dimensional arguments to yield similar results.

Our other major assumption is that only unbiased measurement errors enter into the determination of the intrinsic binary period decay rate \dot{P}_b . The determination of \dot{P}_b requires an accurate distance measurement to the binary system, however, which can be difficult to make. The large uncertainty in the discrepancy Δ associated with PSR B1534+12 may well be due to an underestimate of the distance to this binary system [7], in which case the bound on m^2 would be even tighter.

The bound described here arises from the properties of dynamical relativity, making it conceptually independent of either the solar system or galactic cluster bounds on the graviton mass, which are based on the Yukawa form of the static field in a massive theory. Furthermore, the bounds from any given pulsar system will improve as observations increase the accuracy of the measured fractional discrepancy in the period derivative.

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TABLES

TABLE I. Orbital parameters and corresponding graviton mass bound from the two binary pulsar systems whose gravitational wave induced orbital decay has been measured. Pulsar parameters are taken from [6,7]. One-sigma uncertainties are quoted for Δ .

	PSR B1913+16	PSR B1534+12
Period	27907 s	36352 s
Eccentricity	0.61713	0.27368
Δ	$0.32\% \pm 0.35\%$	$-12.0\% \pm 7.8\%$
Graviton mass 90% upper bound	$9.5 \times 10^{-20} \text{ eV}/c^2$	$6.4 \times 10^{-20} \text{ eV}/c^2$

FIGURES

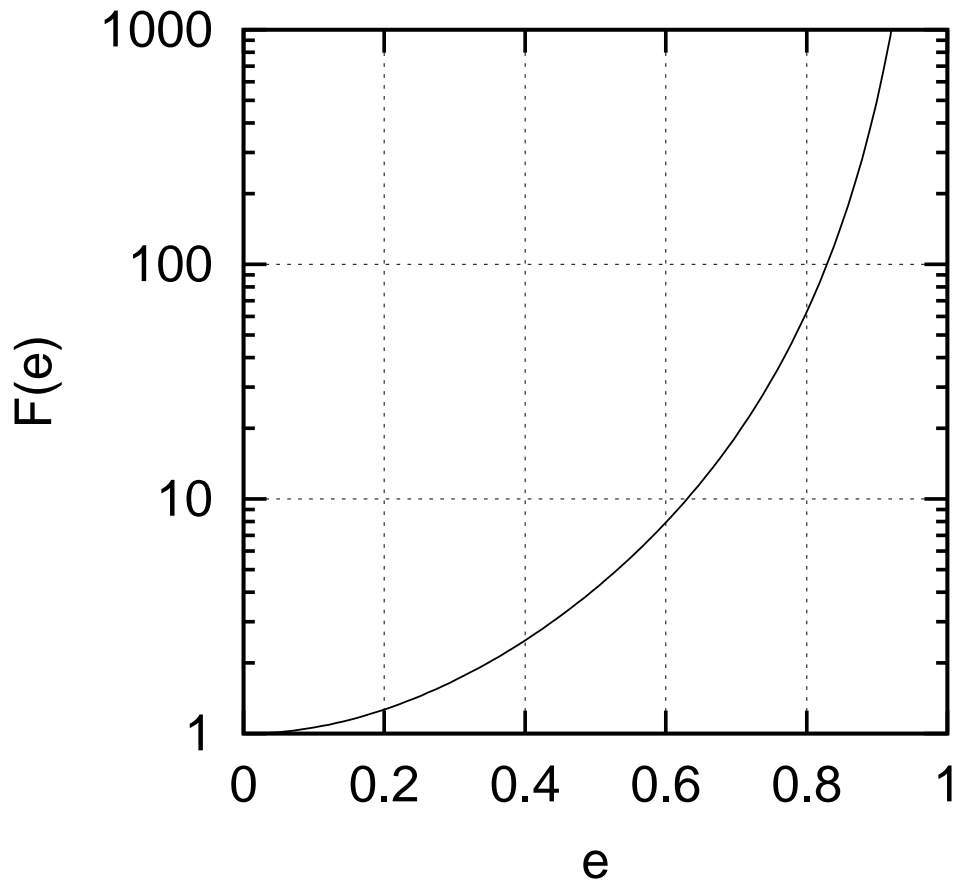


FIG. 1. Eccentricity factor $F(e)$ (cf. eqn. 4.3) versus e .