Statistical characteristics of a stochastic background of gravitational waves from neutron star formation

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Abstract. By using a recent model for the evolving star formation rate, we investigate the statistical distribution of gravitational wave amplitudes due to supernovae that result in neutron star formation in the Einstein–de Sitter cosmology. To account for the uncertainty in gravitational wave emission for this source type, we use a random mixture of three simulated waveform types computed by Zwerger & Müller. We investigate statistical parameters of the resulting gravitational wave amplitude distribution in our frame.

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1. Introduction

Coward, Burman & Blair (2001a) calculated the spectral strain due to a background of gravitational waves (GWs) from neutron star (NS) formation in supernovae (SNe) at cosmological distances, using a sample of three single-source GW waveforms from a catalogue calculated by Zwerger & Müller (1997, hereafter ZM). By using the evolving star formation rate (SFR) models of Madau et al. (1998) and the Einstein–de Sitter cosmology, a differential rate of NS formation as a function of redshift was obtained. By integrating this differential rate with the GW single-source spectra, the corresponding strain spectra from a stochastic background of SNe that lead to NS formation was calculated.

In this paper we use a Monte Carlo method developed by Coward et al. (2001b), in order to simulate a background of GWs due to a cosmological ensemble of discrete GW sources. The amplitude distribution of the events follows a probability distribution function, $p(z)$, that is based on the differential rate of events as a function of redshift z, with z treated as a random variable. For a standard single-source waveform, the random variable z determines the amplitude and duration of each event as observed in our frame; the duration follows directly from z and the amplitude from the cosmology-dependent luminosity distance. Hence the amplitude distribution of an ensemble of events, as observed in our frame, is related to the particular cosmology and SFR model used.

2. The differential NS formation rate

As the progenitors of NSs are short-lived massive stars with lifetimes of some tens of Myr, the NS formation rate will closely track the evolving star formation rate. To account for the variation of this rate with the evolution of the SFR we use a rate density evolution factor, $e(z)$. We base this on a high-dust-extinction SFR model, ψ, in which half of the present-day stars were formed at $z = 2$–5 and were enshrouded by dust (Madau et al. 1998):

$$\Psi(t) = 0.336 \exp(-t_9/1.6) + 0.0074 \exp[1 - \exp(-t_9/0.64)] + 0.0197 {t_9}^5 \exp(-t_9/0.64) \quad \mathrm{M_\odot\ yr^{-1}\ Mpc^{-3}} \tag{1}$$

where t_9 is the time in Gyr since the Big Bang; this model was derived using the Einstein–de Sitter cosmology with Hubble constant $H_0 = 50$ km s^{-1} Mpc^{-1}, so $t_9 = 13(1+z)^{-3/2}$. We assume a cutoff for star formation at $z = 5$, because active star formation is believed to have begun at that epoch. The dimensionless evolution factor $e(z)$ is defined as $\psi(z)/\psi(0)$.

The variation of the rate of NS formation with redshift can be expressed by the event rate equation

$$dR/dz = 4\pi(c^3 r_0/{H_0}^3)e(z)F(z)/(1+z) \tag{2}$$

where r_0 is their present-epoch rate density and $R(z)$ is the all-sky event rate, as observed in our local frame, for sources out to redshift z. The factor $c^3 r_0/H_0{}^3$ has the dimensions of inverse time, as does the event rate $R(z)$. The $(1+z)$ denominator in (2) accounts for the time dilation of the observed rate by cosmic expansion, converting a source-count equation to an event rate equation. The dimensionless function $F(z)$ is determined by the cosmological model; for the Einstein–de Sitter cosmology (Peebles 1993, p. 332):

$$F(z) = 4(1+z)^{-5/2}\left[2 + z - 2(1+z)^{1/2}\right] . \tag{3}$$

Following Rapagnani (1990) we take the local rate density of SNe of all types to be 1×10^{-11} SNe s^{-1} Mpc^{-3}, corresponding to about 1 event per year within 10 Mpc. Surveys of nearby galaxies suggest that 65% of SNe in typical spiral galaxies (Capellaro et al. 1999) are the result of core collapse. Assuming that each core collapse results in either a NS or a BH and using a Salpeter initial mass function, we calculate the branching ratio of these event types to be about 4:1. A local rate density of NS births of 6×10^{-12} NS s^{-1} Mpc^{-3} results.

3. Simulating the GW background

Normalizing the differential rate of NS formation converts it to a probability distribution function with z treated as a random variable (Coward et al. 2001b):

$$p(z) = \frac{dR}{dz} \div \int_0^5 \frac{dR}{dz} dz . \tag{4}$$

The corresponding cumulative distribution function $C(z)$, giving the probability of an event occurring in the redshift range 0 to z ($z \leq 5$), is

$$C(z) = \int_0^z p(x)dx. \tag{5}$$

We use the inverse of $C(z)$ to produce values of z by employing a random number generator to select values from $C(z)$.

The amplitudes of standard GW bursts, as measured on Earth and originating from SNe at cosmological distances, will vary with z by being inversely proportional to the cosmology-dependent luminosity distance, $d_L(z)$; for the Einstein–de Sitter cosmology,

$$H_0 d_L(z)/c = 2(1+z)\left[1 - (1+z)^{-1/2}\right] . \tag{6}$$

The duration of the waveform will be time-dilated by the factor $(1+z)$ owing to cosmological expansion.

The amplitude, for a standard source, is completely determined by the random variable z through $d_L(z)$. Combining this with Poisson-distributed arrival times t_i in our frame and the time-dilation factor $(1+z)$, the i^{th} GW waveform in a time series can be expressed in our frame as

$$h_i(t) = h\left(\frac{t - t_i}{1+z}\right) \frac{(10\ \mathrm{Mpc})}{d_L(z)} , \qquad i = 1, 2, 3, ..., \tag{7}$$

where the waveform $h(t)$ can be any generic time-limited GW signal for a source at a fiducial distance of 10 Mpc.

The mean temporal interval between successive GW events (in this case NS births) is the inverse of the total cumulative event rate (in our frame) throughout the Universe. This quantity and the typical time-dilated waveform duration will determine the duty cycle. A high duty cycle will mean that overlap between successive waveforms will be common. So we calculate the time series as a sum of waveforms to account for interference between successive GW events:

$$f(t) = \sum_i h_i(t) \ . \tag{8}$$

4. Numerical results

The three simulated GW waveforms used in this study, chosen from a set of 78 core-collapse simulations by Zwerger and Müller (1997), are classified as Type I, II and III. Each type is distinguished by the adiabatic index of the core material. The different core-collapse scenarios are reflected in the distinctly different single-source GW spectra. Type I is characterized by core bounce and ring-down, and Type II is characterized by distinct spikes which result from the core bouncing several times. Type III has a large positive and smaller negative wave amplitudes just before and after core bounce.

Using the above technique, we have simulated the background of GWs from sources at cosmological distances, taking into account cosmological and evolutionary effects. To account for variability in the GW emission among individual sources, we use a random mixture of 3 waveforms representative of Types I, II and III from ZM. For this simulation, the rate of events, calculated in our frame, is about 35 s^{-1} (Coward et al. 2001b), with a mean duration for each event of order milliseconds.

Figure 1 shows a 22-s time trace, with a sampling frequency of 2 kHz (corresponding to 44,000 data points) of the resulting dimensionless strain amplitude in our frame. Figure 2 is a histogram of this time trace, showing that the amplitude distribution is sharply peaked; the skewness (3rd moment about the mean normalized to the cube of the standard deviation) is about -3 and the kurtosis (4th moment about the mean normalized to the 4th power of the standard deviation) is about 40. These statistics correspond to the highly non-Gaussian nature of this type of GW background. The kurtosis value shows how sharply peaked the distribution is compared with Gaussians, which have kurtosis 3.

The statistical characteristics of a background will depend on the rate of events as observed in our frame. As the rate of events increases, the central limit thereom tells us to expect that the amplitude distribution will approach a Gaussian, with a mean and variance determined by the event rate. Our results indicate that the statistical characteristics of the NS-formation background will be highly non-Gaussian, given the total rate of NS formation calculated previously (Ferrari et al. 1999, Coward et al. 2001a). So this background will be fundamentally different from the primordial GW background, which is expected to be Gaussian.

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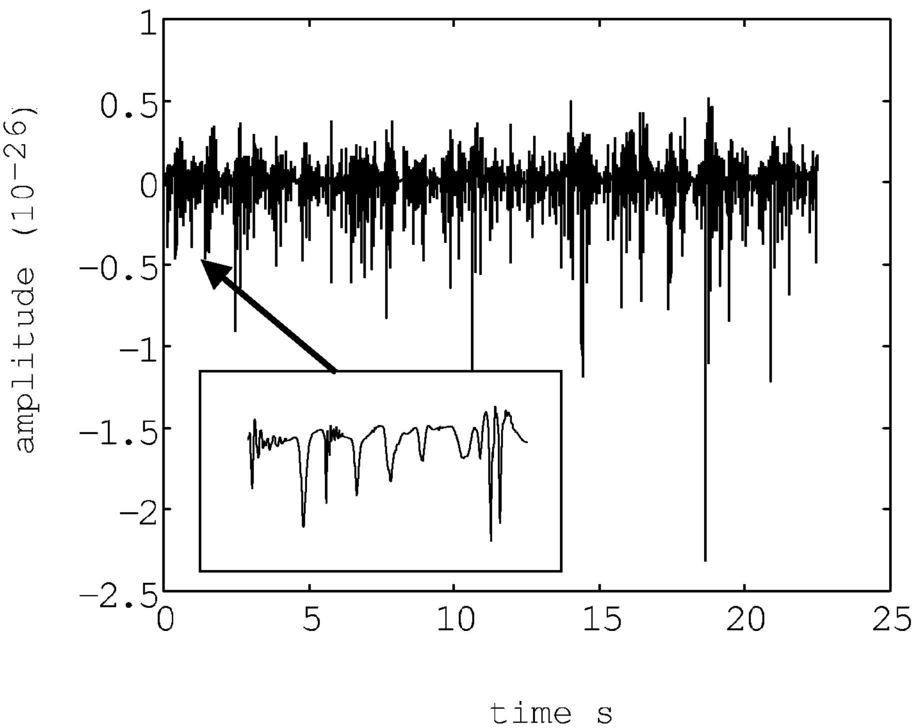

Figure 1. A 22-s time trace using a random selection of three GW waveforms representative of Types I, II and III from Zwerger and Müller (1997). The sampled time series was obtained using a sampling frequency of 2 kHz. The inset is the first 0.5 s.

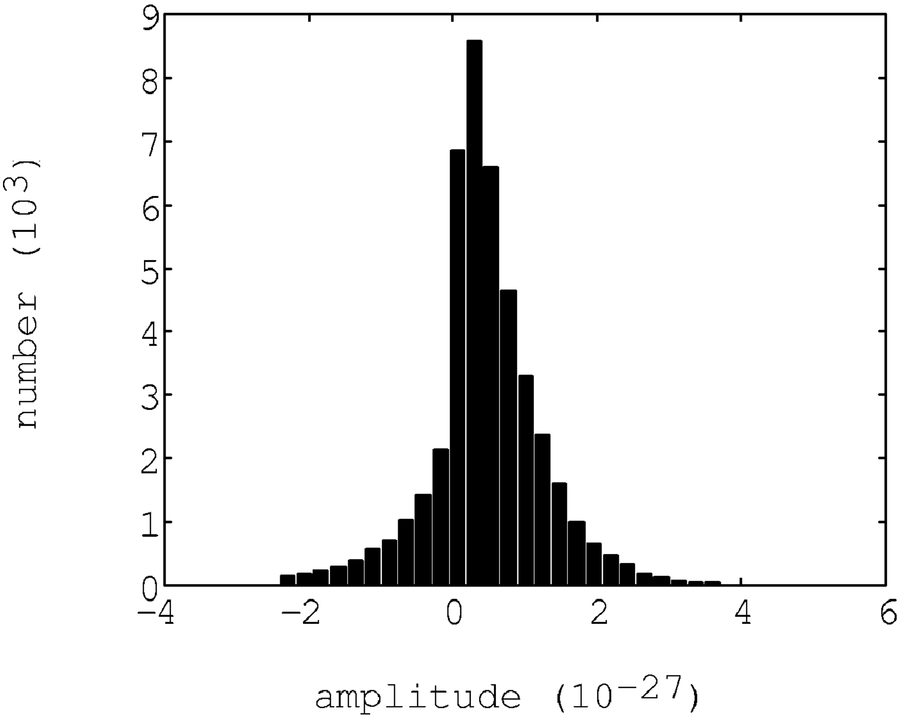

Figure 2. A histogram of the 22-s time series showing the amplitude distribution. It is skewed and sharply peaked, indicating a non-Gaussian distribution.