An Adaptive Optics Approach to the reduction of misalignments and beam jitters in Gravitational Waves Interferometers

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Abstract. We describe a study and preliminary experimental results on the possibility of using adaptive optics systems for the reduction of geometrical fluctuations of input laser beam in long baseline interferometric detectors of gravitational waves. The experimental tests aimed to test the efficiency of Hermite-Gauss versus Shack-hartmann wavefront reconstruction and feed-back diagonalization. These preliminary results seem to indicate that adaptive optics systems may be integrated in the next future as a stabilization stages before a passive mode cleaner cavity, provided that the operational band of the mirror is increased together with the efficiency of the control system.

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1. Introduction

Many very long baseline interferometers detectors for gravitational waves are becoming operational in these years in the world, with bands in the range from 10 Hz up to some kHz and sensitivities better than $10^{-21}\, m/\sqrt{Hz}$[1][2][3][4]. In order to reach these goals, the residual phase noise of such interferometers must be of the order of $10^{-11}\frac{Hz}{\sqrt{Hz}}$ in the measurement band. And, among all the noises that can contribute to increase such phase noise, the noise due the coupling between geometrical fluctuations of input laser beam and interferometers asymmetries is of great importance [5].

In fact, interferometer asymmetries correspond to different fundamental modes of the two cavities which can be described by writing the fundamental mode of each cavity as a proper sum of the fundamental and higher order modes of the ideal interferometer. In the same way, the input beam geometrical fluctuations can be regarded as fluctuations of higher order modes of the beam with respect to the reference cavity mode [6][7][8]. It can be demonstrated that interferometer asymmetries corresponding to the higher order modes generate phase difference by coupling with laser beam geometrical fluctuations corresponding to the quadrature of the same mode and the resulting noise. If typical free laser geometrical fluctuations are considered, this would impose requirements on the maximal allowed interferometric asymmetries, impossible to fulfill with the present technology [8].

The most widely chosen solution for the present generation of interferometers is that of reducing this noise by reducing the geometrical fluctuations of the input laser beam *passively*, by means of a mode-cleaner. In fact, the input laser beam passes through a Fabry-Perot cavity of Finesse of few hundreds ($F = 1000$ in Virgo) before entering the interferometer. Geometrical fluctuation are seen by the cavity as higher order modes, which do not resonate in the cavity. These modes are then reflected, producing in this way a noise suppression of the same order of the cavity Finesse [5][9][10][11]. With the help of this auxiliary cavity, the requirements on interferometer symmetry can be relaxed but, whenever present technology should allow to fulfill these requirements, they still remain very stringent. As an example, residual angular laser jitters fluctuations, corresponding to the first Hermite-Gauss mode, impose that the optical axis of a Virgo arm, for example, will be controlled better that $\delta\theta < 10^{-9}\, rad$, while waist position fluctuations and astigmatism impose limits of $\delta W/W_0 < 5 \times 10^{-3}$, on cavity waist asymmetry and $\delta R/R_0 < 10^{-2}$, where δR is the difference in mirror radius and R_0 the mean radius.

Actually, the above requirements can be fulfilled with the present technology, even if we furtherly include cavity changes due to thermal effects. Nevertheless, a more stable input beam could relax these requirements, allowing large improvements in the robustness of an apparatus which is expected to work continuously on a timescale of years.

A better passive beam stabilization with improved Finesse mode-cleaner presently seems a difficult task. An alternative possibility is the use of hybrid techniques based on both active and passive fluctuations reduction. A first stage, based on an Adaptive Optics system followed by a passive Fabry-Perot cavity seems a good compromise, with very promising results. Moreover, the adaptive optics system could be useful to correct also long term geometrical fluctuations, typically due to thermal deformations of cavities and matching optics, resulting in a better beam-cavity matching and consequent lower light losses.

In this paper we present preliminary results on a beam geometrical fluctuations reduction using a commercial Deformable Mirror, with wavefront reconstruction and error signal generation based on the classical Shack-Hartmann technique. The aim of the experiment is to verify the possibility of an easy Hermite-Gauss modes reconstruction and to test the robustness of the real-time feed-back system. This experiment can be considered as a first step toward the demonstration and eventually the design and integration of an Adaptive Optics system within gravitational interferometers.

2. Wavefront analysis: Gaussian beam perturbations and Zernike polynomials

In order to use the Shack-Hartmann phase front reconstruction for recovering Gaussian beam perturbations, expressed a as sum of Hermite-Gauss modes, it is convenient to write a generic beam perturbed in waist position, different in the two axis, corresponding to a defocus and astigmatism starting from the x, y of the Shack-hartmann axes. If x' and y' are the two principal astigmatic axes, corresponding to maximum and minimum curvature radius, it is possible to write

$$x' = x\cos\overline{\theta} + y\sin\overline{\theta} \qquad y' = y\cos\overline{\theta} + x\sin\overline{\theta} \tag{1}$$

and the field of the perturbed beam as

$$\Psi = \frac{A}{w_0}\exp\left[-\frac{x'^2+y'^2}{w_0^2} - i\frac{\pi w_0^2}{\lambda R_{x'}}\frac{x'^2}{w_0^2} - -i\frac{\pi w_0^2}{\lambda R_{y'}}\frac{y'^2}{w_0^2}\right] \tag{2}$$

where $R_{x'}$ e $R_{y'}$ are the curvature radii and

$$\frac{1}{R} = \frac{1}{2}\left(\frac{1}{R_{x'}} + \frac{1}{R_{y'}}\right) \tag{3}$$

In this expression R is the mean radius, expressing the defocus, while

$$\delta\left(\frac{1}{R}\right) = \frac{1}{2}\left(\frac{1}{R_{x'}} - \frac{1}{R_{y'}}\right) \tag{4}$$

is the astigmatism. Using then the relations

$$\frac{1}{R_{x'}} = \frac{1}{R} + \delta\left(\frac{1}{R}\right) \qquad \frac{1}{R_{y'}} = \frac{1}{R} - \delta\left(\frac{1}{R}\right) \tag{5}$$

the field Ψ can be written as

$$\Psi = \frac{A}{w_0}\exp\left[-\frac{x'^2+y'^2}{w_0^2} - i\frac{\pi w_0^2}{\lambda}\left(\frac{1}{R} + \delta\left(\frac{1}{R}\right)\right)\frac{x'^2}{w_0^2} - i\frac{\pi w_0^2}{\lambda}\left(\frac{1}{R} + \delta\left(\frac{1}{R}\right)\right)\frac{y'^2}{w_0^2}\right] \tag{6}$$

By using $x'^2 + y'^2 = x^2 + y^2$, then the field Ψ can be written as

$$\Psi = \frac{A}{w_0} \exp\left[-\frac{x'^2 + y'^2}{w_0^2} - i\frac{\pi w_0^2}{\lambda R}\left(\frac{x'^2 + y'^2}{w_0^2}\right) - i\frac{\pi w_0^2}{\lambda}\delta(\frac{1}{R})\left(\frac{x'^2 - y'^2}{w_0^2}\right)\right] \tag{7}$$

Then, by using the relation

$$x'^2 - y'^2 = x^2(\cos\overline{\theta}^2 - \sin\overline{\theta}^2) - y^2(\cos\overline{\theta}^2 - \sin\overline{\theta}^2) + 4xy\cos\overline{\theta}\sin\overline{\theta} \tag{8}$$

and some algebra we can finally write

$$\Psi = \frac{A}{w_0} \exp\left(-\frac{x'^2 + y'^2}{w_0^2}\right)\Big[1 - i\frac{\pi w_0^2}{\lambda R}\left(2\rho^2 - 1\right) - i\frac{\pi w_0^2}{\lambda}\delta(\frac{1}{R})(\rho^2)\cdot$$

$$\cdot(2\cos\theta^2 - 1)(\cos\overline{\theta}^2 - \sin\overline{\theta}^2) - i\frac{\pi w_0^2}{\lambda}\delta(\frac{1}{R})4\cos\overline{\theta}\sin\overline{\theta}\rho^2\cos\theta\sin\theta\Big] \tag{9}$$

This this last equation show that if the wavefront is analyzed on the waist surface then there is a direct correspondence between Hermite-Gauss modes of the field and wavefront Zernike polynomials. In using Shack-Hartmann wavefront reconstruction, is thus convenient to analyze the wavefront on the region up to the beam waist. In fact, the scalar product between wavefront and Zernike polynomials directly gives the coefficient for the corresponding Hermite-Gaussian mode. In particular, the coefficient obtained with the zernike polynomial Z_4 (defocus) is $C_{Z_4} = \frac{\pi w_0^2}{\lambda R}$ corresponding to the coefficient of the mode $\frac{1}{\sqrt{2}}(U_{02} + U_{02})$. The coefficient of 45-Astigmatism is $C_{Z_5} = \frac{\pi w_0^2}{\lambda}\delta(1/R)4\cos\overline{\theta}\sin\overline{\theta}$ corresponding to the mode U_{11}. Finally, the coefficient of the 90-astigmatism is $C_{Z_6} = \frac{\pi w_0^2}{\lambda}\delta(1/R)(\cos\overline{\theta}^2 - \sin\overline{\theta}^2)$ corresponding to the mode $\frac{1}{\sqrt{2}}(U_{02} - U_{02})$.

We stopped the analysis to these coefficients because only the modes of order 2, together angular jitters, are of particular importance in gravitational wave interferometers. In a future work a complete analysis of all modes will be carried out.

3. Experimental apparatus and results

The efficiency of Hermite-Gauss versus Shack-Hartmann wavefront reconstruction and feed-back diagonalization, actuator efficiency and feed-back robustness have been tested with the experimental apparatus schematically shown in Figure 1.

The beam is first expanded to match about 2/3 of the deformable mirror diameter, then by some optics is sent to a Shack-Hartmann grid and reduced to match the CCD camera, characterised by $15\mu m$ pixels, which allows a sensitivity of the order of $1.5\mu m$. This implies a sensitivity of about 0.01 in the evaluation of the Hermite-Gauss higher order mode coefficients and a sensitivity of about $1\mu rad$ for the first mode. Then, in order to recover the wavefront just on all the waist surface, the camera is software masked according to the analysis described in the previous section. The correction actuator is a commercial Micro Machined Deformable Mirror, produced by OKO-Flexible Technology, $1.5\,cm$ diameter, actuated with 37 electrostatic actuators. This device has been chosen

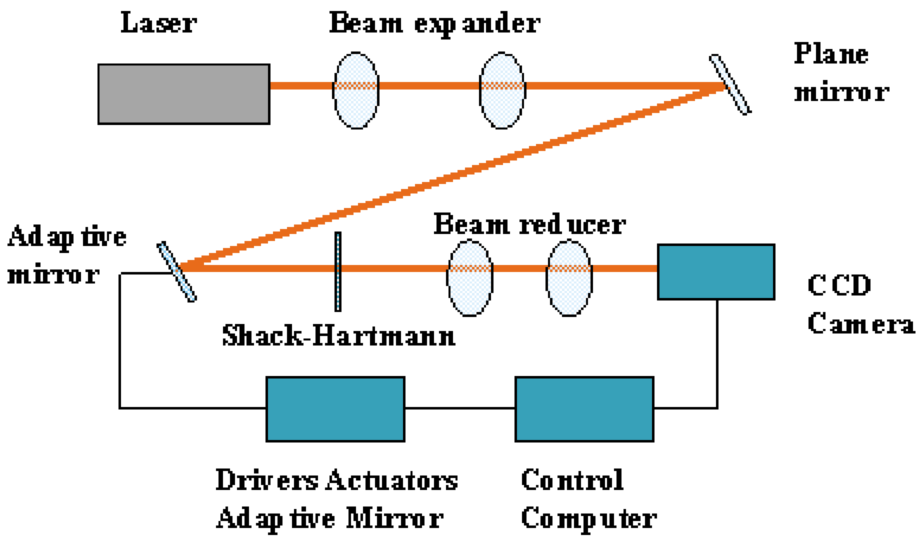

Figure 1. Adaptive Optics System Experimental Apparatus

Table 1. Initial and final value of wavefront perturbations expressed as tilts and coefficients of Hermite-Gauss modes (tilts are expressed directly in radians).

Modes	TiltX	TiltY	$\frac{1}{\sqrt{2}}\,(U_{02}+U_{02})$	U_{11}	$\frac{1}{\sqrt{2}}\,(U_{02}-U_{02})$
Open	215μ rad	$634\mu rad$	0.7	6×10^{-2}	0.3
Closed	0.2μ rad	0.6μ rad	3×10^{-3}	2×10^{-2}	2×10^{-2}

for his low cost, efficiency and robustness. The laser is a commercial $10\,mW$ He-Ne. In this first test the wavefront is analyzed by 36 points (lens), while the error and correction signals, obtained by a simple low pass filter of the error signal, are real-time evaluated using a PC. The sampling frequency is thus low, few Hz, and the unity gain of the feed-back is about $1\,Hz$.

In order to perform a test on feed-back efficiency, the wavefront has been initially perturbed, while the effect of the feed-back is shown in the error signal of the corrected wavefront (see Figure 2). In Table 1 the initial value of tilts, defocus and astigmatism are shown. This initial wavefront corresponds to a careful manual micrometric alignment, while the final wavefront is of the order of the expected sensitivity, showing that the high order mode analysis, the error signal diagonalization and the resulting feed-back are robust and efficient, with a gain above 100 at low frequency.

4. Conclusions

We have described a preliminary test to demonstrate the possibility of using adaptive optics for the correction of laser beam in gravitational wave interferometers, aiming at

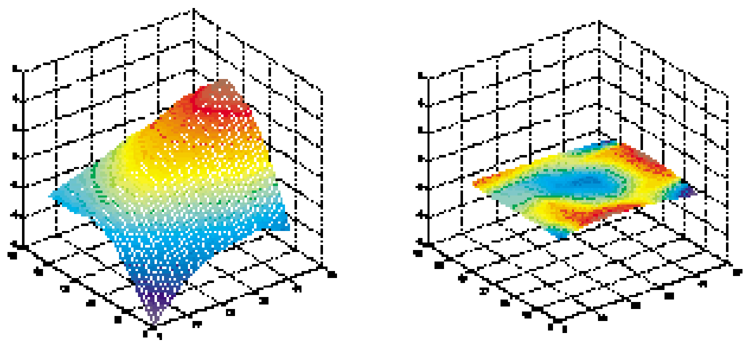

Figure 2. Initial and Corrected Wavefront in unit of wavelength

the relaxation of the presently stringent limits on interferometer allowed by geometrical asymmetries.

The experimental test has regarded both the efficiency of Hermite-Gauss versus Shack-Hartmann wavefront reconstruction, feed-back diagonalization, and actuator efficiency and robustness. Results show the efficiency of this system, whenever important the next steps must be, and its capability of performing the operations at a speed that although not too fast, can be improved by changing the feedback section.

If this integration and subsequent tests on deformable mirror efficiency at higher frequency will be positive, then adaptive optics could be integrated in the interferometer as a stabilization stage before the passive mode-cleaner cavity, allowing a safer interferometer operation.

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