IGEC toolbox for coincidence search

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Abstract. The standard IGEC approach to detection of gravitational waves with many detectors is simple time coincidence search. We discuss the problems of false alarm and false dismissal assessment, both in the case of stationary and non-stationary noise. The significance of any cumulative excess of found coincidences over the background is determined by maximum likelihood methods.

1. Introduction

Gravitational waves (gw) resonant detectors operating so far are narrow band detectors, and their typical bandwidths are no more than a few Hz with a central frequency of about 900 Hz [1]. Therefore, they are insensitive to any structure present in the waveform of the impinging gw, they just sample the amplitude of the Fourier component of the signal at a few discrete frequencies. The impulse response template is routinely used as a first approximation to describe all burst classes whose duration is of the order of one millisecond.

The search for bursts is performed by threshold crossing (the signal overcome a certain amplitude threshold)[2, 3, 4] or maximum-hold (the signal is locally at maximum amplitude)[5] methods. This search usually allows very low ($\sim 3 \div 5$) signal-to-noise ratios (SNRs). Estimating false alarms by relying on the nominal probability of local fluctuation of a Gaussian stochastic process is often not viable, because outliers (for instance, due to environmental transient noise) are unavoidable, and they mimic the effect of GW signals. The puzzle can be solved when two or more detectors are working at comparable sensitivity. Given that the delay between successive triggers is much longer than the time uncertainty on the arrival time of the burst, the chances of an accidental coincidence are proportionally low, the more the detectors the better.

In this paper we report on coincidence search analysis as it is carried on within the International Gravitational Event Collaboration (IGEC)[1, 6]. First we deal with an operative definition of the concept of coincidences. In section 2 we shall briefly describe how this definition is linked to estimates of timing uncertainty and to detection efficiency. We shall discuss the issue of accidental coincidences in section 3. The maximum likelihood criterion will eventually guide us to the assessment of detection confidence (section 4).

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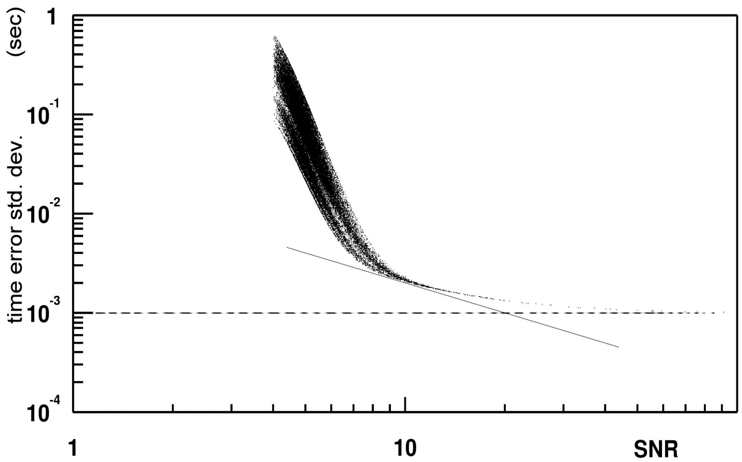

Figure 1. Standard deviation of the time error vs. SNR for triggers exchanged within IGEC by the AURIGA collaboration(courtesy of AURIGA group). The dashed horizontal line represent the lower bound due to present systematic calibration errors, the continuous line is the asymptotic scaling of the standard deviation as $1/SNR$ expected at high SNR. Notice that 88% of the events have SNR up to 6. Spreading at low SNR is due to variations of the Wiener filter bandwidth.

2. Triggers, coincidences and time windows

The basic ingredients of the gw search recipe of IGEC are the self-consistent "*event files*", which are temporally ordered records of every *event* (or *trigger*) selected by each group of the collaboration as candidate gw signals [1]. It is understood that, before the event search, the data are processed by a Wiener filter matched to the impulse response of the system, what we call a $delta-like$ GW. The parameters that fully describe such signal are just the estimated time of arrival (ETA) and amplitude together with their error statistics, and —when available— a test statistic to assess compliance of the signal with the template. Bundled in the same exchanged files, there are asynchronous information about amplitude precision and accuracy, and the values of the amplitude selection threshold. Despite the slightly different event search algorithm applied, for our purpose it is sufficient to recall that a dead time is associated with every trigger, usually bigger than the time standard deviation of the time error, and of the order of the inverse of the Wiener filter bandwidth.

The sequence of ETA in a single event list can be described by a (possibly non- homogeneous) Poisson point series, which requires the triggers to be *rare* and *independent*. A coincidence is defined as a Mtuple of triggers, one for each detector, with ETA such that there is a common overlap between their *time windows*. More precisely, if $\mathbf{t}^{(m)}$ is the set of ETAs from detector labelled m, the following relation defines the set of all coincidences:

$$\left\{ \boldsymbol{c}_n = (t_1, t_2, \ldots, t_M) | \, t_m \in \mathbf{t}^{(m)} \& \, \forall h, k : |t_h - t_k + \Delta t_{hk}| < (\Delta t_h + \Delta t_k) \right\}_{n=1,2,\ldots} \quad (1)$$

where Δt_m is half the length of the time window associated with the event occurring at time t_m in the mth detector, and Δt_{hk} corrects for time delay in signal propagation at light speed between the hth and kth detectors. Δt_m is a function of the probability density of the timing error once a minimum confidence level (or a maximum

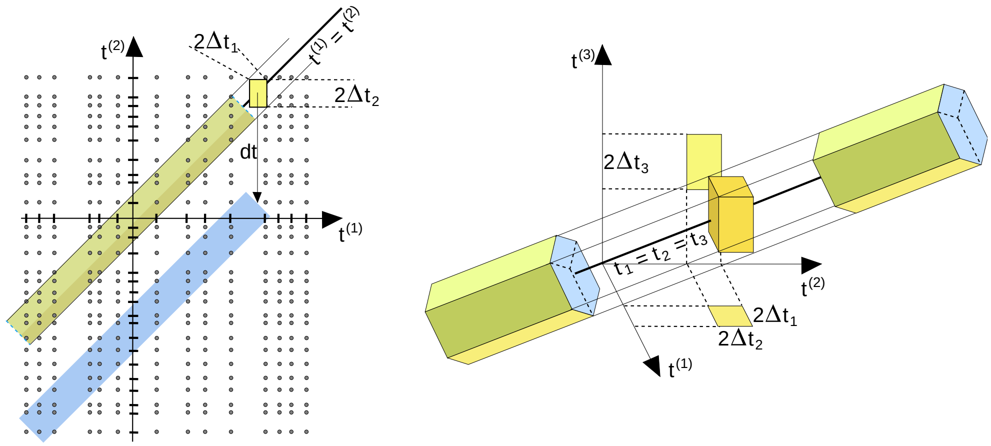

Figure 2. (*left*) The set of all possible pairings between two triggers in a two-detectors configuration can be represented with a lattice in 2-dim plane, obtained by tensor product of the original single detector event lists. In the simplified case when every trigger of one detector has the same time window, coincidence search can be seen as a particular selection of events in a stripe (*fiducial volume*) along the bisector, which corresponds to synchronous detector timescales. A time-shifted search means to count trigger pairs that fall inside a new translated stripe. (*right*) In a similar way, in a three-detectors configuration, the geometrical representation of the fiducial volume for coincidence search is a tube obtained developing the three dimensional error box along the bisector.

false dismissal probability) has been fixed. This relation is non-analytical and depends on the implementation of trigger selection and on the SNR of the signal, and it has to be determined by Monte Carlo methods. Figure 1 shows a typical scaling of the time error standard deviation σ with respect to SNR. The exact relation between σ_m and Δt_m depends on the robustness of the model for the statistics of the noise. The Gaussian model would give $\Delta t_m = 1.96\sigma_m$ —for a confidence level $c = 95\%$— but in general this does not apply. Using the very general Bienaymè-Tchebicheff inequality, $c \leq 95\%$ if $\Delta t_m = \sigma_m/\sqrt{1-c} \approx 4.5\sigma_m$.

If the source direction is not available, its determination and consistency test on arrival time sequence is deferred to a later analysis, and the relation $|t_h - t_k + \Delta t_{hk}| < (\Delta t_h + \Delta t_k)$ in (1) is substituted by $|t_h - t_k| < (\Delta t_h + \Delta t_k + \max \Delta t_{hk})$, allowing the maximum delay between the labelled pair of detectors.

3. False alarms estimates

A simple analytic formula for the expected background rate of accidental coincidences with the definition (1) is viable by choosing a fixed value for the standard deviation of the timing noise of each detector. This approximation is only slightly worse than the optimal one, as most of the events has amplitudes just above the thresholds (see figure 1)

If the background triggers can be modelled as a homogeneous Poisson point process, and being $\lambda^{(k)}$ and Δt_k respectively the background rate and time window of the kth (over M) detector, the expected false alarm rate λ_b is given by†

$$\lambda_b = C_M(\Delta t_1, \ldots, \Delta t_M) \times \prod_{i=1}^{M} \lambda^{(k)} \tag{2}$$

where

$$C_M(\Delta t_1, \ldots, \Delta t_M) \equiv \sum_{k=1}^{M} \prod_{h \neq k} 2\Delta t_h \tag{3}$$

Equation (2) holds also as *instantaneous* background rate predictor, when the background is non-stationary, and as linear operations (like integrals) on Poisson random variable (RV) give as a result a Poisson RV, this leads to

$$\lambda_b(t) = C_M(t) \times \prod_{i=1}^{M} \lambda^{(k)}(t) \quad \Leftrightarrow \quad \bar{\lambda}_b = \frac{1}{T} \int_{t=0}^{T} dt\, C_M(t) \prod_{i=1}^{M} \lambda^{(k)}(t) \tag{4}$$

In practical use of this result, the observation time is divided in smaller time intervals where the background process can be considered stationary, but long enough to estimate the statistics of the detector noise parameters.

A numerical and more accurate method to estimate false alarms probability is time-shifted search strategy. It consists in generating independent detector configurations by adding small delays to the ETA of all but one detector trigger lists. These configurations are supposed to be independent when the delay is applied in steps longer than the maximum total time coincidence window: ergodicity of the system is then required. In fact, referring to figure 2, the counts of coincidences inside each translated stripe are in principle different RV, but we use them as independent instantiations of the counts inside the synchronous stripe.

The time shift analysis allows to estimate the false alarm rate even when the velue of the coincidence window varies from trigger to trigger, to deal with variations of SNR and of the Wiener filter bandwidth. Moreover it naturally keeps care of the non-homogeneous character of the trigger rate, at least up to delays of the same order of magnitude as the smallest-scale time structure in the data. It is therefore suited for non-clustered data sets. As the previous analytic method, the estimated number of accidentals coincidences is still biased by the assumption that the event list contains no real signal, but since the latter are much more rare, this effect is totally negligible. Finally, we remark that, by repeated independent time shifts, the Poisson statistics of the coincidence counts can be directly tested, instead of being blindly assumed.

† To figure out how this formula can be derived, the reader should refer to figure 2, where the cases M=2 and M=3 are depicted. The measure of the fiducial volume is the product of its transverse section by its length. The first factor is given by the measure of the $M-1$ dimensional surface obtained by projecting the M-dim error box in the plane transverse to the bisector. This means to multiply $1/\sqrt{n}$ to the measure $\prod_{h\neq k} 2\Delta t_h$ of each hyper-face, and summing over all directions $k = 1, ..., M$. The second factor is equal to the common observation time of the M detectors divided by the same quantity $1/\sqrt{n}$, which then cancels out in the product. In order to get the number of accidental coincidences, the measure of the fiducial volume have to be multiplied by the density of the lattice nodes representing each possible coincidence, which is given by $\prod_{i=1}^{M} \lambda^{(k)}$. Dividing by the common observation time gives at last (2).

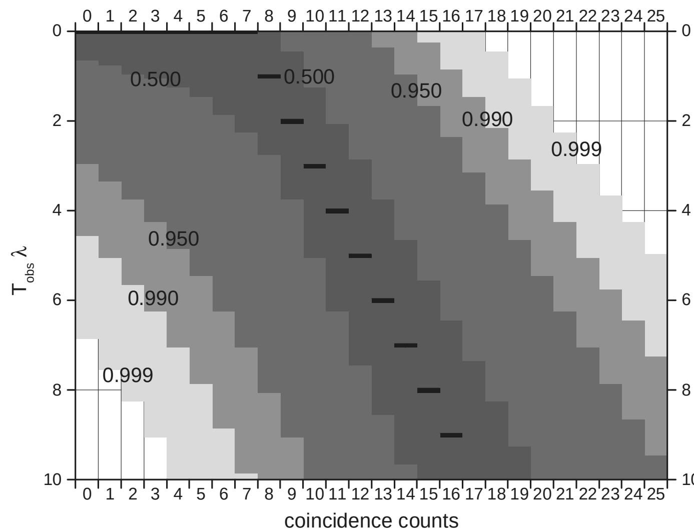

Figure 3. Example of confidence level (CL) contour plot for a background $\mathbf{N}_b = 7$. For each possible outcome (i.e. coincidence count, $\mathbf{N}_c$) in the abscissa, the corresponding confidence interval can be determined by finding along the vertical axis the intercepts with a pair of contours, enclosing the desired CL (in this example, the 50%, 95%, 99% and 99.9% CL contours are plotted). Up to 12 counts the null hypothesis is still allowed at 95% confidence level, while above 12 counts it is ruled out. The thick black lines indicate the most likely value for the number of GW.

4. Computation of average gw burst rate

At last, when the number of predicted and found coincidences have been determined, a decision has to be made about the presence among the latter of gw signals. In the usual frequentist approach, a straightforward result in terms of confidence intervals is obtained by maximum likelihood methods.

Given the mean number of expected background coincidences $\bar{\mathbf{N}}_b \equiv E\{\mathbf{N}_b\}$ over a time T_{obs}, let Λ be the unknown mean rate of triggers produced by random (and rare) sources, which is natural to model with a Poisson point process. Even if it happens to be non-homogeneus, the number of coincidences N_c counted in a typical experiment is a statistical sample of a Poisson RV $\mathbf{N}_c$ with mean $\bar{\mathbf{N}}_c = \bar{\mathbf{N}}_b + T_{obs}\Lambda$. Let it be $N_\Lambda \equiv T_{obs} \cdot \Lambda$. The probability density function of $\mathbf{N}_c$ computed at the observed value is

$$P_{N_c}(\bar{\mathbf{N}}_b + N_\Lambda) \equiv \frac{1}{N_c} \left(\bar{\mathbf{N}}_b + N_\Lambda\right)^{N_c} e^{-\left(\bar{\mathbf{N}}_b + N_\Lambda\right)} \tag{5}$$

and the corresponding likelihood function is

$$\ell(N_\Lambda; N_c, \bar{\mathbf{N}}_b) \equiv P_{N_c}(N_\Lambda + \bar{\mathbf{N}}_b) \tag{6}$$

with the obvious bound $N_\Lambda > 0$. The most likely value of N_Λ supported by the observations is obtained when ℓ is maximum, and for any *confidence level* (CL) $c \in [0,1]$, its associated *confidence interval* $[N_{\inf}, N_{\sup}]$ is defined by $\ell(N_{\inf}) = \ell(N_{\sup})$ and $\int_{N_{\inf}}^{N_{\sup}} \ell(N)dN = c$. We shall say that the number of found coincidences N_c "*agrees at confidence level c with a rate Λ between $N_{\inf}/T_{obs}$ and $N_{\sup}/T_{obs}$* ". in the case

$N_{\rm inf} = 0$ the null hypothesis is not ruled out by the experiment, and we shall say "$N_{\rm sup}/T_{obs}$ is the upper limit at confidence level c". The frequentist interpretation of this statement is that the actual outcome is $c/(1-c)$ times more likely to be observed over many repetitions of the experiment if the value of N_Λ is inside the interval $[N_{\rm inf}, N_{\rm sup}]$, than if it were outside.

Figure 3 shows an example of confidence intervals set at different found coincidences, with the same value of estimated background accidentals $\bar{\mathbf{N}}_b = 7$. At 95% CL, up to $N_c = 12$, no positive detection claim is allowed, but only upper limits.

5. Conclusions

Within IGEC, the presently used coincidence search method varies the time window aperture in order to guarantee a specific maximum false dismissal. The corresponding false alarm statistics is then empirically investigated by assuming the ergodic approximation and taking time-shifted configurations of the observatory as independent instantiations of the observations at zero time delay. Finally, by feeding the standard maximum likelihood test with the estimated false alarm rates, the significance of cumulative excess of coincidences can be assessed.

References

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